Supporting Information


Quasi-Three-Dimensional Angle-Tolerant Electromagnetic Illusion Using Ultrathin Metasurface Coatings

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Department of Electrical Engineering, The Pennsylvania State University, University Park, PA 16802, USA
E-mail: zuj101@psu.edu, dhw@psu.edu

**Retrieving \( \vec{\alpha}_E \) and \( \vec{\alpha}_M \) of the metasurface**

When the metasurface coating is wrapped around a cylinder, the surface polarizability tensor parameters \( \alpha_{E}^{\phi} \), \( \alpha_{E}^{\varphi} \), and \( \alpha_{M}^{p} \) are active under TM\(_z\) polarized plane wave excitation. Since each unit cell is subwavelength (< \( \lambda_0/14 \)) in the \( \varphi \) direction, the metasurface can be considered as locally flat. Hence, when designing the unit cell of the metasurface, the planar configuration (see Figure S1(a)) can be adopted where the active surface polarizability tensor parameters are \( \alpha_{E}^{x} \), \( \alpha_{M}^{x} \), and \( \alpha_{E}^{y} \), which are equivalent to \( \alpha_{E}^{\phi} \), \( \alpha_{M}^{p} \), and \( \alpha_{E}^{\varphi} \) shown in Figure 2(e), respectively.

**Figure S1.** Metasurface illusion coating unit cell for (a) the dielectric cylinder, *i.e.* the first example and (b) the conducting cylinder, *i.e.* the second example.
When the cylinder to be coated is made of Teflon, as in the case for our first example, the metasurface property is not affected by the presence of the Teflon due to its low relative permittivity \((i.e.\) only 2.1\). Therefore, a standard material parameter extraction scheme can be utilized as discussed in [34]. Specifically, in the HFSS simulation domain, periodic boundary conditions are assigned on the lateral walls, while Floquet ports are employed at a certain distance above and below the metasurface to excite the incident plane wave. To retrieve \(\alpha^x_E\) and \(\alpha^z_M\), transverse electric (TE) polarized plane waves at two different incidence angles are required, which generate the complex reflection and transmission coefficients \(r_1(\theta_1), r_1(\theta_2), t_1(\theta_1),\) and \(t_1(\theta_2),\) respectively. Hence, the plane of incidence coincides with the \(y-z\) plane while the incident electric field is aligned in the \(x\) direction. The parameters \(\alpha^x_E\) and \(\alpha^z_M\) can then be extracted via the following expressions:

\[
\begin{align*}
\alpha^x_E &= \frac{(2\cos\theta_2 \sin^2\theta_2 - r_1(\theta_2)^2 - 2r_1(\theta_2) - 2t_1(\theta_2)^2 - 2r_1(\theta_2)^2)}{jk_0} \frac{1 + r_1(\theta_1)^2 + t_1(\theta_1)^2 + 2t_1(\theta_1)}{1 - r_1(\theta_1)^2 + t_1(\theta_1)^2 + 2t_1(\theta_1)} \\
\alpha^z_M &= \frac{(2\cos\theta_1 \sin^2\theta_1 - r_1(\theta_1)^2 - 2r_1(\theta_1) - 2t_1(\theta_1)^2 - 2r_1(\theta_1)^2)}{jk_0} \frac{1 + r_1(\theta_2)^2 + t_1(\theta_2)^2 + 2t_1(\theta_2)}{1 - r_1(\theta_2)^2 + t_1(\theta_2)^2 + 2t_1(\theta_2)}.
\end{align*}
\]  

(S1a)

(S1b)

To retrieve the parameter \(\alpha^y_E\), a TM polarized plane wave at normal incidence is required with its incident magnetic field aligned in the \(x\) direction, which generates the complex reflection coefficient \(r_2(0^\circ)\). Hence, \(\alpha^y_E\) can be determined from

\[
\alpha^y_E = \left(\frac{1}{jk_0}\right)\frac{-2r_2(0^\circ)}{r_2(0^\circ) + 1},
\]

(S2)

When the cylinder to be coated is comprised of copper, as in the case of our second example, then the metasurface property will be altered due to the near-field coupling between the metasurface and the conductive copper cylinder in close proximity. In order to correctly extract the superficial electromagnetic properties of the metasurface in this case, a copper backing at a distance \((a_c - a_d)\) below the metasurface is employed (see Figure S1(b)). Specifically, in the
HFSS simulation domain, in addition to the periodic boundary conditions assigned on the lateral walls, only a single Floquet port is used at a certain distance above the metasurface to excite the incident plane wave. A dielectric spacer made of foam is inserted in between the metasurface and the copper backing. To retrieve $\alpha_E^x$ and $\alpha_M^x$, TE polarized plane waves at normal and oblique angles of incidence are required, which generate complex reflection coefficients $r_1(0^\circ)$ and $r_1(\theta_i)$, respectively. The plane of incidence lies in the y-z plane while the incident electric field is polarized along the x direction. The parameters $\alpha_E^x$ and $\alpha_M^x$ can then be extracted from

\[
\begin{align*}
\alpha_E^x &= \frac{1}{j k_0} \left( \frac{1-r_1(\theta_i=0^\circ)}{r_1(\theta_i=0^\circ)+1} - \frac{1+e^{-j2k_0d_s}}{1-e^{-j2k_0d_s}} \right), \\
\alpha_M^x &= \frac{1}{\sin^2\theta_i} \left( \frac{\cos\theta_i}{j k_0} \left( \frac{1-r_1(\theta_i)}{r_1(\theta_i)+1} - \frac{1+e^{-j2k_0d_s\cos\theta_i}}{1-e^{-j2k_0d_s\cos\theta_i}} \right) - \alpha_E^x \right),
\end{align*}
\]

where $k_s$ is the wave number of the spacer in between the metasurface and the copper backing, while $d_s = (a_c - a_d)$ is the thickness of this spacer. To retrieve $\alpha_E^y$, a TM polarized plane wave at normal incidence is required with its incident magnetic field aligned in the x direction, which generates a complex reflection coefficient given by $r_2(0^\circ)$. Hence, $\alpha_E^y$ can be extracted by using the following expression:

\[
\alpha_E^y = \frac{1}{j k_0} \left( \frac{1-r_2(\theta_i=0^\circ)}{r_2(\theta_i=0^\circ)+1} - \frac{1+e^{-j2k_0d_s}}{1-e^{-j2k_0d_s}} \right),
\]
Metasurface coated dielectric cylinder under a cylindrical wave excitation

Figure S2. Snapshots of the full-wave simulated total $E$-field magnitude distribution for (a) the targeted copper cylinder, (b) the uncoated Teflon cylinder, and (c) the same Teflon cylinder coated by the designed metasurface under a cylindrical wave excitation at a distance $d_{cyl} = 0.5 \lambda_0$. (e-g) The same set of plots with the cylindrical wave excitation at a distance of $d_{cyl} = 1 \lambda_0$. The normalized far-field power patterns with (d) $d_{cyl} = 0.5 \lambda_0$ and (h) $d_{cyl} = 1 \lambda_0$. 


3D RCS of finite-length Teflon cylinder with metasurface-enabled illusion coating

Figure S3. 3D RCS patterns of the targeted finite-length copper cylinder with (a) $\theta_{tilt} = 0^\circ$ and (b) $\theta_{tilt} = 10^\circ$. 3D RCS patterns of the finite-length Teflon cylinder with (c) $\theta_{tilt} = 0^\circ$ and (d) $\theta_{tilt} = 10^\circ$. 3D RCS patterns of the finite-length metasurface-coated Teflon cylinder with (e) $\theta_{tilt} = 0^\circ$ and (f) $\theta_{tilt} = 10^\circ$. All the RCS patterns are normalized to the maximum value of the simulated RCS of the targeted copper cylinder at normal incidence. The plane wave excitation is propagating in the +x direction with its electric field along the z-direction. With the metasurface coating, the 3D RCS patterns of the finite-length Teflon cylinder are nearly identical to those of the targeted finite-length copper cylinder except for the presence of minor diffraction lobes. The dashed lines represents the horizontal (x-y) plane.
Metasurface coated copper cylinder under a cylindrical wave excitation

Figure S4. Snapshots of the full-wave simulated total $E$-field magnitude distribution for (a) the targeted Teflon cylinder, (b) the uncoated copper cylinder, and (c) the same copper cylinder coated by the designed metasurface under a cylindrical wave excitation at a distance $d_{\text{cyl}} = 0.5 \lambda_0$. 
(e-g) The same set of plots with the cylindrical wave excitation at a distance $d_{\text{cyl}} = 1 \lambda_0$. The normalized far-field power patterns with (d) $d_{\text{cyl}} = 0.5 \lambda_0$ and (h) $d_{\text{cyl}} = 1 \lambda_0$. 
3D RCS of finite-length copper cylinder with metasurface-enabled illusion coating

Figure S5. 3D RCS patterns of the targeted finite-length Teflon cylinder with (a) $\theta_{tilt} = 0^\circ$ and (b) $\theta_{tilt} = 10^\circ$. 3D RCS patterns of the finite-length copper cylinder with (c) $\theta_{tilt} = 0^\circ$ and (d) $\theta_{tilt} = 10^\circ$. 3D RCS patterns of the finite-length metasurface-coated copper cylinder with (e) $\theta_{tilt} = 0^\circ$ and (f) $\theta_{tilt} = 10^\circ$. All the RCS patterns are normalized to the maximum value of the simulated RCS of the targeted Teflon cylinder at normal incidence. The plane wave excitation is propagating in the $+x$ direction with its electric field along the $z$-direction. With the metasurface coating, the 3D RCS patterns of the finite-length copper cylinder are nearly identical to those of the targeted finite-length Teflon cylinder except for the presence of minor diffraction lobes. The dashed lines represent the horizontal ($x$-$y$) plane.