A.1 INTRODUCTION

Solver is an Excel add-in for optimization, which involves finding the values of a set of decision variables that maximize or minimize an objective function. A basic version of Solver comes with every copy of Excel, but more advanced and powerful versions are available from Frontline Systems (www.solver.com).

A.2 CONCEPTS

Four concepts are essential in optimization:

- Decision variable
- Objective function
- Constraint
- Linearity

A decision variable is an input value chosen by the decision maker in a problem represented by a model. Typical examples include the number of employees to hire, the amount of raw materials to buy, or the price to charge for a service.

An objective function is a mathematical formula that calculates the result the decision maker uses to determine how well their choices work out. Examples include profit, NPV, or cost. The objective function is either maximized or minimized. Thus, one would want to maximize profit but minimize cost.

A constraint is a restriction or requirement that decision variables must meet. Typical constraints include a budget that cannot be exceeded, or a requirement that at least 25 percent of a portfolio is invested in stocks.

Linearity refers to the mathematical form of the objective function and constraints. Linear functions in this context involve nothing more complex.
than multiplying decision variables by constants and adding up. So, for example, the following budget constraint is linear:

\[
\text{Budget} \geq \text{Travel expenses} + \text{Software expenses} + \text{Phone expenses}
\]

Likewise, the following objective function is linear:

\[
\text{Profit} = 32 \times \text{Product A Sales} + 45 \times \text{Product B Sales} + 55 \times \text{Product C Sales}
\]

**A.3 ALGORITHMS**

Solver uses a variety of mathematical approaches to find the optimal values of the decision variables. These methods are known as algorithms. Three main algorithms are available in most versions of Solver:

- LP Simplex
- GRG Nonlinear
- Evolutionary

The LP Simplex method is used strictly for problems in which the objective function and all constraints are linear. Essentially, it solves different subsets of constraints to find the combination of decision variables that optimizes the objective function.

The GRG Nonlinear method will work on linear problems, but it is designed for problems with nonlinear objectives, nonlinear constraints, or both. (A problem is considered nonlinear for Solver if the objective function or even one constraint is not linear.) It takes the existing values of the decision variables residing in the spreadsheet as its initial solution and considers small changes in those variables that improve the objective. In this way, it gradually marches “uphill” if the goal is to maximize, or “downhill” if the objective is to minimize, until it reaches an optimal solution. Because this so-called “hill-climbing” search procedure cannot see the entire mountain it is trying to climb, it is possible for it to get stuck on a small peak while a higher peak sits in the distance. This is the problem of local optima, which can be dealt with by trying different starting values for the decision variables and testing whether different final solutions are reached.

The Evolutionary method uses the genetic algorithm approach to find optimal or near-optimal solutions. In this approach, a group (or population) of solutions is generated and this population is then subjected to random mutation and natural selection. Good solutions—those with high values of the objective if the goal is to maximize or low values if the goal is to minimize—are preserved from generation to generation, whereas bad solutions are killed off. In this way, the fitness of the population improves over time and eventually the best remaining solutions will be optimal or near optimal.
Figure A.1 shows a simple advertising planning model in which the goal is to maximize profits by choosing how much to advertise in each of the four quarters of the year. The decision variables are Advertising Expenditures in cells D18:G18. The objective is to maximize Total Profit in cell C21. There are two sets of constraints in the problem. One constraint is that advertising expenditures cannot be negative. The other is that the total amount spent on advertising (cell H18) must be less than the advertising budget (cell C15).

To invoke the Solver add-in, go to Add-ins—Premium Solver. The Solver Parameters window shown in Figure A.2 will open. The “Set Target Cell” is the objective function in cell C21. Choose “Equal To: Max” to maximize this cell. Enter the cell addresses of the decision variables (D18:H18) in the window.
under “By Changing Cells.” Finally, the budget constraint is entered by choosing Add and then by entering H18 \leq C15. We can ensure that the decision variables are not negative by choosing Options and checking the box for “Assume Non-Negative.” Make sure “Assume Linear Model” is not checked. Click on OK to return to the Solver Parameters window.

Choose Solve to run Solver with these specifications. A few seconds later the window in Figure A.3 will appear, signifying the successful termination of the algorithm and offering the choice to keep the original values of the decision variables in the spreadsheet or to keep the Solver solution.

Figure A.4 shows the model after optimization. The optimal advertising plan involves high expenditures in quarters 2 and 4 and low expenditures in quarters 1 and 3. Total profit has increased from $69,600 to $71,447.

The objective function in this model is nonlinear. We can demonstrate this by creating a graph of Profit as a function of, say, Q1 Advertising. Or we can examine the formula in row 27, where we see that the model assumes decreasing returns to advertising. Had we chosen the option “Assume Linear Model,” Solver would have detected the nonlinearity and returned the warning message shown in Figure A.5. (Note: different versions of Solver have somewhat different inputs and windows.)
**Figure A.4.** Optimized version of Advertising Budget model.

**Figure A.5.** Solver warning when model is not linear.
We often want to know how the optimal solution to a problem changes as we vary an input parameter. For example, in the advertising budget example discussed above, we might ask how the profit would change if we could increase the budget. This requires running Solver for a variety of different budget values. This can be done by changing the budget input, running Solver, and recording the results—a tedious procedure if we want to test a large number of inputs. The Solver Sensitivity tool in the Sensitivity Toolkit can be used to automate this process. For more information on using this tool, see Appendix C.

The results of a Solver Sensitivity run are shown in Figure A.6. We varied the Budget in cell C15 from $40,000 to $100,000 in increments of $5,000 (column A). Solver was run for each of these 13 cases and the optimal value of the objective function reported in column B. Finally, the unit change in the objective is calculated in column C. We see that Profit increases (at a decreasing rate) with the Budget up to about $90,000, beyond which it no longer increases.

### References