**A**

**Equal Area Criterion**

This is a direct method to check the stability of a synchronous generator (connected to an infinite bus) that is subjected to disturbance(s). The method does not require the solution of the nonlinear equation. The assumptions in applying the equal area criterion (EAC) are:

1) classical machine model described by the swing equation
2) rotor damping is neglected.

The application of the EAC assumes that if the system is stable in the first swing, it continues to remain stable although the rotor may oscillate about its final steady state.

Consider the swing equation given by

$$M \frac{d^2 \delta}{dt^2} = T_m - T_e \approx P_m - P_e = P_a$$

(A.1)

where $M = \frac{2H}{\omega_p}$ is the inertia, $\delta$ is the rotor angle with respect to the synchronously rotating reference, and $P_m$ and $P_e$ are the mechanical and electrical (output) power, respectively. It is assumed that the operating speed is assumed to be at rated value (1 pu).

Multiplying both sides of (A.1) by $\frac{d\delta}{dt}$ and integrating with respect to time ($t$), we get

$$M \int_{t_o}^{t} \frac{d\delta}{dt} \cdot \frac{d^2 \delta}{dt^2} \, dt = \int_{t_o}^{t} P_a \frac{d\delta}{dt} \, dt$$

(A.2)

This is simplified to

$$\frac{1}{2} M \left( \frac{d\delta}{dt} \right)^2 = \int_{t_o}^{t} P_a \, d\delta$$

(A.3)

It is assumed that at $t = t_o$ the system is at rest (in equilibrium state) and $\frac{d\delta}{dt} = 0$. The right-hand side of the above equation can be interpreted as the area between the curves, $P_m$ versus $\delta$ and $P_e$ versus $\delta$. If the system is to be stable, then

$$\frac{d\delta}{dt} \bigg|_{t=t_p} = 0, \quad t_p > t_o$$

(A.4)

It can be observed that $\delta = \delta_{max}$ at $t = t_p$ and $\delta$ starts reducing for $t > t_p$. Assuming that there is some damping (both electrical and mechanical), then the oscillations will finally die out and the rotor angle will reach an equilibrium which may be identical to $\delta_o$ or a new equilibrium which depends on the post-disturbance system.


K. R. Padiyar and Anil M. Kulkarni.

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From (A.4), we note that the area defined by

$$A = \int_{\delta_0}^{\delta_{\text{max}}} (P_m - P_e) \, d\delta$$

(A.5)

must have a positive portion $A_1$ for which $P_m > P_e$, and a negative $A_2$ for which $P_m < P_e$. For the system to be transiently stable the two areas $A_1$ and $A_2$ must be identical in magnitude and $A = A_1 - A_2 = 0$. Hence, the nomenclature of EAC. This is illustrated in Figure A.1 assuming $P_m$ is raised from $P_{m0}$ to $P_{m1}$ (at $t = t_o$ when $\delta = \delta_o$).

Remarks

1) The EAC is also applicable for a two-machine system (without an infinite bus) as it can be converted into a single-machine equivalent (SIME).

2) Mathematically, the problem of determination of transient stability can be viewed as checking whether the initial system state ($\delta_o$) lies in the region of stability surrounding the post-disturbance stable equilibrium. It should be noted that every stable equilibrium point (SEP) has a region of stability in which a trajectory approaches SEP as $t \to \infty$. A trajectory starting outside the region of stability will not approach SEP and may even be unbounded.

3) EAC is a special case of the direct method for stability evaluation using energy functions [1].

An Interesting Network Analogy [2]

Consider a circuit consisting of a linear capacitor connected to a nonlinear inductor and excited by a DC current source $I$ (see Figure A.2).

The initial current in the inductor is $I$ and the initial voltage across the capacitor is $v_0$. The nonlinear inductor is defined by

$$i = I_m \sin \lambda$$

(A.6)

where $\lambda$ is the flux linkage of the inductor. The equations for this circuit are given by

$$C \frac{dv}{dt} = I - i, \quad \frac{d\lambda}{dt} = v$$

(A.7)
Equal Area Criterion

Figure A.2 A linear capacitor connected to a nonlinear inductor.

\[ i \quad C \quad + \quad - \quad \\upsilon \]

The energy associated with the capacitor is \( \frac{1}{2} CV^2 \) while the change in energy of the inductor is given by

\[
\Delta W_L = \int_{\lambda_0}^{\lambda} i \, d\lambda \quad \text{(A.8)}
\]

Substituting from (A.6), we get

\[
\Delta W_L = I_m [\cos \lambda_0 - \cos \lambda] \quad \text{(A.9)}
\]

The variation of the current with \( \lambda \) is sinusoidal, as shown in Figure A.3. The initial value of \( \lambda = \lambda_0 \) is given by

\[
\lambda_0 = \sin^{-1} \left( \frac{I}{I_m} \right) \quad \text{(A.10)}
\]

The maximum increase in energy of the inductor is given by the area ABCDE in Figure A.3 (which shows current in the inductor as a function of flux linkages). This is given by (A.9) when we substitute \( \lambda = \lambda_u = \pi - \lambda_0 \).

However, the constant DC current source \( I \) is contributing energy \( \Delta W_S \) given by

\[
\Delta W_S = I(\lambda_u - \lambda_0) \quad \text{(A.11)}
\]

Note that the maximum energy transfer from capacitor to inductor is only \( \Delta W_L - \Delta W_S \). If this energy is greater than the initial energy stored in the capacitor \( \left( \frac{1}{2} CV_0^2 \right) \), the capacitor discharges completely to the nonlinear inductor and the maximum flux linkage \( (\lambda_m) \) is less than or equal to \( \lambda_u \). The oscillation of the energy between the capacitor

Figure A.3 Current vs flux linkage.
and the inductor continues indefinitely unless damped by dissipation in resistors in the circuit (which have been neglected in this analysis). If \( \frac{1}{2} C v_{0}^2 > (\Delta W_L - \Delta W_S) \) then the inductor cannot fully absorb the initial energy in the capacitor and \( \lambda \) continues to increase, implying the capacitor voltage continues to increase.

It can be shown that this example is similar to the determination of transient stability in a lossless single-machine infinite bus (SMIB) system. The SMIB system can be modeled by a linear capacitor (analogous to the machine inertia) connected to a nonlinear inductor (analogous to the net series reactance of the system connected between the generator internal bus and the infinite bus). The loss of synchronous stability in the SMIB system is analogous to the failure of the capacitor to completely transfer the stored energy to the nonlinear inductor. The current in the inductor is analogous to the power flow in the line and the voltage across the capacitor is analogous to the deviation in the rotor speed of the generator following a disturbance. Thus, the stored energy in the linear capacitor is analogous to the kinetic energy of the generator rotor moving with respect to a synchronously rotating reference frame.

The network analogy helps in deriving the total energy for detailed, structure-preserving system models. Apart from direct evaluation of system stability, the energy functions can also be used for online detection of loss of synchronism and the mode of instability accurately. In addition, it becomes feasible to determine accurately the optimum locations and the control laws for the network (flexible AC transmission system) controllers for enhancing system stability [2].

References