Testing slope effect on flow resistance equation for mobile bed rills

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Abstract
In this paper, a recently theoretically deduced rill flow resistance equation, based on a power-velocity profile, is tested experimentally on plots of varying slopes in which mobile bed rills are incised. Initially, measurements of flow velocity, water depth, cross-sectional area, wetted perimeter and bed slope conducted in 106 reaches of rills incised on an experimental plot having a slope of 14% were used to calibrate the flow resistance equation. Then, the relationship between the velocity profile parameter τ, the channel slope, and the flow Froude number, which was calibrated using the 106 rill reach data, was tested using measurements carried out in plots having slopes of 22% and 9%. The measurements carried out in the latter slope conditions confirmed that (a) the Darcy–Weisbach friction factor can be accurately estimated using the proposed theoretical approach, and (b) the data were supportive of the slope independence hypothesis of rill velocity stated by Govers.

KEYWORDS
flow resistance, plot measurements, rill hydraulics, soil erosion, velocity profile

1 | INTRODUCTION

Rill development is controlled by the detachment of soil particles and sediment transport by the channelized flow, and rill hydraulics may differ greatly from the hydraulics of flow in larger channels (Foster, Huggins, & Meyer, 1984). In fact, rills are small and ephemeral flow paths that act as sediment sources and transport both rill flow-detached particles and those delivered from the interrill areas (Bagarello & Ferro, 2004, 2010; Bagarello, Di Stefano, Ferro, & Pampalone, 2015; Bruno, Di Stefano, & Ferro, 2008; Di Stefano, Ferro, Pampalone, & Sanzone, 2013; Di Stefano, Ferro, & Pampalone, 2015). Furthermore, flow depths in rills are typically of the order of millimetres to several centimetres and bed topography, characterized by steep slope values, significantly affects flow hydraulics (Abrahams, Li, & Parsons, 1996; Foster et al., 1984; Gilley, Kottwitz, & Simanton, 1990; Govers, 1992; Nearing et al., 1997; Peng, Zhang, & Zhang, 2015).

Rill erosion modelling requires that rill flow has to be adequately modelled (Abrahams et al., 1996) because the hydraulic conditions are very different from those typically found in stream and river open channel flow (Nearing et al., 1997).

Rill channels are also actively eroded and their morphology evolves at a small temporal scale. A given rill morphology is the result of changes in channel width, flow depth, bed roughness due to the eroding bed material, the flow discharge, and the initial rill structure.

The interaction between flow, rill channels, sediment transport creates the morphology and affects the evolution of the erosion process.

Notwithstanding available experimental data (Govers, 1992; Peng et al., 2015; Takken, Govers, Ciesiolka, Silburn, & Loch, 1998) suggesting that the use of Chezy, Manning, and Darcy–Weisbach empirical flow resistance formulas (Ferro, 1999; Powell, 2014) may be questioned for rill flows, many of the physically based soil erosion models currently apply fundamental theories of river open channel flows (Govers, Giménez, & Van Oost, 2007; Nouwakpo et al., 2016; Takken et al., 1998) in the absence of a well-developed theory for rill flow hydraulics (Peng et al., 2015).

When a theoretical velocity distribution (Baiamonte, Ferro, & Giordano, 1995; Ferro, 2003) can be applied to the velocity profiles measured in different verticals of the cross section of flow (Ferro & Baiamonte, 1994), the integration along the cross section will theoretically allow one to deduce the flow resistance law. The unavailability of velocity measurements is surrogated by the use of empirical flow resistance formulas (Ferro, 1999; Powell, 2014) such as:

$$V = C \sqrt{R s} = \frac{s^{1/2} R^{2/3}}{n} = \sqrt{\frac{8 g R s}{f}} \tag{1}$$

in which V is the cross-sectional average velocity, C is Chezy coefficient (m^{1/2} s^{-1}), n is Manning coefficient (m^{-1/3} s), f is dimensionless friction factor, s is the slope and Rs is the hydraulic radius (m).
Darcy–Weisbach coefficient, s is channel slope, R is hydraulic radius, and g is acceleration due to gravity.

The effect of slope gradient on flow velocity and roughness in rills is a topic subjected to scientific debate (Nearing et al., 1997).

Foster et al. (1984) carried out a laboratory study using a full-scale fiberglass, fixed bed, replica of a rill formed on an erosion plot having slopes equal to 3%, 6%, and 9%. They proposed a velocity-discharge-slope relationship that shows that the flow velocity increases as a function of slope gradient.

Govers (1992) analysed 409 measurements on mean flow velocities in rills collected by several authors and concluded that rill flow velocity tends to be independent of slope. Govers (1992) hypothesized a feedback mechanism between flow erosivity and bed morphology that should be responsible of slope independency of flow velocity (Giménez & Govers, 2001). The expected increase of flow velocity due to slope gradient is counterbalanced by the effect of the increase of erosion rate with increasing slope. This last effect produces an increase of bed roughness, thereby slowing the flow velocity.

Torri, Poesen, Borselli, Bryan, and Rossi (2012) also supported that the effect of slope on mean flow velocity seems to be compensated by the effects of increased erosion and increased bed roughness resulting in a mean flow velocity being controlled by total discharge alone. The increase of roughness with erosion rate was also confirmed by rill erosion experiments conducted by Xinlan, Longshan, Jia, and Faqi (2015).

The general accepted concept is that bed-load extracts momentum from the flow determining the flow velocity reduction and the increase of the apparent roughness length (Baiamonte & Ferro, 1997). Song, Chiew, and Chin (1998) investigated the effect of bed-load movement on the friction factor and concluded that particle motion increases flow resistance. Recking, Frey, Parquier, Belleudy, and Champagne (2008), using experimental runs with and without bed-load transport, concluded that bed-load maximizes flow resistance and a roughness parameter 2.5 times higher than for clear water has to be used. Therefore, the absence of a slope effect on rill flow velocity is due to a feedback mechanism between rill bed morphology and flow conditions (Giménez & Govers, 2001; Govers et al., 2007). The absence of a slope effect on rill flow velocity was also reported by Nearing et al. (1997), Nearing, Simanton, Norton, Bulygin, and Stone (1999) and by Takken et al. (1998), who carried out experimental runs with rills incised on an unconsolidated soil.

Assuming as correct the slope independence hypothesis of rill velocity, the use of a constant hydraulic roughness coefficient for equations such as Darcy–Weisbach Equation 1 could be inappropriate for eroding rills. In fact, Equation 1 allows one to obtain a slope-independent rill flow velocity only if the value of Manning’s n (Hessel, Jetten, & Guanghui, 2003) or the Darcy–Weisbach friction factor f is increased with slope gradient.

Because the flow velocity distribution can be expressed by a functional relationship representing the examined physical process, Di Stefano et al. (2017b) suggested that the Π-Theorem of the dimensional analysis and self-similarity theory can be usefully employed to deduce the rill flow resistance equation (Barenblatt, 1979, 1987, 1993; Ferro, 1997).

For a uniform turbulent open channel flow, the velocity distribution along a given vertical can be expressed by the following functional relationship (Barenblatt, 1987, 1993; Di Stefano et al., 2017b; Ferro, 1997):

\[
\Pi = \frac{y}{u^*} \frac{dv}{dy} = \phi \left( \frac{u^* - y}{v_k} \right) \left( u^- h \right) \frac{1}{\nu} \left( \frac{dy}{u^-} \right)
\]

in which v is the local velocity, y is the distance from the bottom, u* = \sqrt{g R} S is the shear velocity, \phi is a functional symbol, h is the water depth, d is the bed particle diameter, and \nu is the water kinematic viscosity.

Assuming the incomplete self-similarity in u*–y/v_k (Barenblatt & Monin, 1979; Barenblatt & Prostokishin, 1993; Ferro, 2017; Ferro & Pecoraro, 2000), Equation 2 allows one to obtain the following velocity distribution:

\[
\frac{v}{u^*} = \Gamma \left( \frac{u^* - y}{v_k} \right)^{\delta}
\]

in which \Gamma is a function to be defined by velocity measurements and \delta is an exponent that can be calculated by the following theoretical equation (Barenblatt, 1991; Castaing, Gagne, & Hopfinger, 1990):

\[
\delta = \frac{1.5}{\ln Re}
\]

in which Re = V h/v_k is the flow Reynolds number.

Integrating the power velocity distribution (Equation 3), the following expression of the Darcy–Weisbach friction factor f is deduced (Barenblatt, 1993; Ferro, 2017; Ferro & Porto, 2017)

\[
f = 8 \left[ \frac{2^{1-\delta} \Gamma \Re^{\delta}}{\left( \delta + 1 \right) \left( \delta + 2 \right)} \right]^{-2/(1-\delta)}.
\]

Setting \gamma = \alpha h, the distance from the bottom at which the local velocity is equal to the cross-sectional average velocity V, from Equation 3, the following estimate \Gamma_v of \Gamma is obtained (Ferro, 2017):

\[
\Gamma_v = \frac{V}{u^- \left( \frac{u^* - \alpha h}{v_k} \right)^{\delta}}
\]

in which \alpha is a coefficient, less than one, taking into account that both the average velocity V is located below the water surface and a single velocity profile representing the whole cross section is considered (i.e., the velocity profile is the mean profile in the cross section, obtained by averaging for each distance y the velocity values v measured in different verticals, and its integration gives the cross-sectional average velocity).

Ferro (2017) theoretically deduced the equation for calculating \alpha and Di Stefano et al. (2017b) established that for 2,000 \leq Re \leq 10,000, which is usually the measurement range used in rill investigations, the calculated \alpha values vary in a very narrow range and a constant value equal to the mean \alpha = 0.124 can be used.

In a previous paper, Di Stefano et al. (2017b) tested the applicability of the flow resistance Equation 5 by some measurements of flow velocity, water depth, cross-sectional area, wetted perimeter, and bed slopes carried out in 106 reaches of some rills shaped on an
The following rough equation for estimating the $\Gamma_v$ function was determined:

$$\Gamma_v = \frac{a F^b}{s^c} \quad (7)$$

in which $F$ is the flow Froude number equal to $V/\sqrt{gh}$ and the coefficients $a = 0.55$, $b = 1.106$, and $c = 0.477$ were estimated by 57 data. Equation 7 with $a = 0.55$, $b = 1.106$, and $c = 0.477$, which was also tested by 49 independent data, is characterized by a coefficient of determination equal to 0.99 and it is applicable for $7% \leq s \leq 19.3%$ and $F$ values ranging from 0.646 to 3.03.

In this study, the relationship given in Equation 7 between the velocity profile parameter $\Gamma_v$, the channel slope, and the flow Froude number, was first recalibrated by using all 106 rill reach data points collected by Di Stefano et al. (2017b) at a slope of 14%. Then the recalibrated Equation 7 was tested using 73 independent measurements carried out in this investigation using plots having slopes equal to 22% and 9%. The measurements carried out in these latter slope conditions were used to confirm that (a) the Darcy–Weisbach friction factor can be accurately estimated by the proposed theoretical approach and that (b) the slope independence hypothesis of rill velocity stated by Govers is reliable for these data.

2 MATERIALS AND METHODS

2.1 Experimental plots

Two experimental plots, located at the experimental area of the Department of Agriculture, Food and Forest Sciences of the University of Palermo, 2 m wide and 7 m long, were used for carrying out the investigation on preshaped rills. The plots, having a slope equal to 9% and 22% (Figure 1), are bounded by concrete walls and have a base constructed of rock gabions. A storage tank was located at the downstream end of the plot, having a width equal to that of the plot and a cross section 0.7 m wide and 0.4 m high. Each plot was filled with a clay soil (73% clay, 22.5% silt, and 4.5% sand). Further details of the experimental installation are reported in Di Stefano et al. (2017a).

2.2 Ground measurement technique

An image-based technique was applied for carrying out the ground measurements of rill cross-sectional area, wetted perimeter, and channel slope. Using a set of oblique photographs (Figure 2) of the rilled surface (Di Stefano et al., 2017a; Frankl et al., 2015), this technique creates a digital terrain model (Gómez-Gutiérrez, Schnabel, Berenguer-Sempere, Lavado-Contador, & Rubio-Delgado, 2014; Westoby, Brasington, Glasser, Hambrey, & Reynolds, 2012; Figure 3). Camera model parameters and scene geometry are simultaneously solved (James & Robson, 2012) using redundant information coming from oblique photographs.

A set of 70 photographs taken from the plot area by a digital camera (Samsung PL210–14.2 Mpx) was captured assuring that all parts of the measured plot area was represented in at least three photographs. This choice allowed us to take into account that the 3D...
the measurement cross section. A correction factor used to convert
the velocity \( V_c \) of the leading edge of the dye cloud to the mean flow
velocity \( V \) equal to 0.8 was used (Li & Abrahams, 1997; Luk & Merz,

In each run, the water depth was measured in the tested cross
sections by a microhydrometer located in the rill thalweg. The wetted
perimeter and the cross-sectional area were calculated by using the
measured water depth in combination with the geometric cross-sectional
profile extracted by digital terrain model. Further details are reported in Di Stefano et al. (2017b).

The water depth \( h \), the hydraulic radius \( R \), and the slope gradient \( s \) of each reach were calculated by averaging the values measured in the
considered reach. The Darcy–Weisbach friction factor \( f_{\text{av}} \) of the reach
was indirectly measured by using Equation 1 with the above measured
values of \( V \), \( R \), and \( s \).

For each experimental run characterized by a known inflow
discharge \( Q \), Table 1 lists the water depth \( h \), slope gradient \( s \), flow
mean velocity \( V \), and measured Darcy–Weisbach friction factor \( f_{\text{av}} \).
The measurements were carried out for the condition both of subcriti-
cal and supercritical flow (flow Froude number \( F = V/\sqrt{g} \) Ti ranged
from 0.74 to 1.9) and fell into transition and turbulent regimes
(Reynolds number \( Vh/v \) varying from 1,306 to 8,537). Figure 4 shows
the experimental (Re, \( F \)) pairs corresponding to both the previous
experiments by Di Stefano et al. (2017b) carried out using a 14% sloped plot and the experimental runs (sp = 9 and 22%) of this
investigation.

### 3 RESULTS AND DISCUSSION

Figure 4a shows the experimental (Re, \( F \)) pairs and demonstrates that
Froude numbers much greater than one were obtained for eroding clay
rills. This result appears in contrast with the statement of Giménez and
Govers (2001) establishing that the feedback mechanism results in a
constant average near-critical Froude number for flow in rills. However, Figure 4b shows that, with the exception of only three
values, the range of the average Froude number \( F_{\text{av}} \) calculated by
averaging the \( F \) values measured in the reaches of a rill, is practically
equal to that (0.9 ≤ \( F \) ≤ 1.4 for 0.28 ≤ \( Q \) ≤ 1.0 L s\(^{-1}\)) investigated by
Giménez and Govers (2001) and the mean value of the average Froude
numbers is equal to 1.05. This comparison is suitable because the
Froude number values calculated by Giménez and Govers (2001) are
also obtained by an averaging procedure of the values measured in
each rill cross section. Finally, present data support the finding of
Giménez and Govers (2001) that, on average, the flow in eroding rills
is critical, whereas the analysis at the reach scale showed that \( F \) values
well above one can be obtained.

The original dataset by Di Stefano et al. (2017b; 106 measure-
ments for the plot having a slope \( sp \) equal to 14%) was used to
calibrate Equation 7, obtaining the following equation:

\[
\Gamma_v = \frac{0.4856 \left(\frac{1.121}{sp^{1.527}}\right)}{\left(\frac{1}{sp^{1.527}}\right)}. \tag{8}
\]

Equation 8 is characterized by a coefficient of determination equal
to 0.99 and it is applicable for flow Reynolds numbers of
2,169 ≤ Re ≤ 10,723, Froude numbers of 0.74 ≤ F ≤ 1.9, and slope s values ranging from 7% to 19.3%.

Figure 5 shows the comparison between the 106 $\Gamma_v$ values calculated by Equation 6 with $\alpha = 0.124$ and those calculated by Equation 8. Equation 8 was also positively tested (Figure 5) by the flow velocity measurements carried out in this investigation ($s = 9$ and 22%), for which $s$ varied from 4.2% to 25.9%. In other words, Equation 8 was also applicable to the measurements corresponding to plots having a slope of 9% and 22%. This result can be justified taking into account that $\Gamma_v$ values corresponding to the plot having a slope 9% (1.52–3.0) and 22% (0.84–2.08) fall within the wide experimental range (0.86–6.05) corresponding to the 14% slope.

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Substituting Equation 4 and Equation 8 into Equation 5, the following flow resistance equation is obtained:

$$f = \frac{8}{2^{1/5} + 0.4856 F^{1.1131} Re^{0.537}} \left(\frac{1.5}{\text{Re}^{0.5}} + 1\right) \left(\frac{1.5}{\text{Re}^{0.5}} + 2\right) s^{0.537} \Gamma_v^{1/5}.$$

(9)

For testing Equation 9 the complete data base of 179 investigated rill reaches was used. Figure 6 shows the good agreement between the measured friction factor values, $f_m$, and the ones, $f$, calculated by Equation 9; this agreement is characterized by a root mean square error equal to 0.044. The friction factor values calculated by Equation 9 are characterized by errors that were less than or equal to ±20% for 95.6% of cases and less than or equal to ±10% for 75.1% of cases.
According to Equation 9, for a given Re value, the Darcy–Weisbach friction factor $f$ increases with slope gradient and decreases when the flow Froude number $F$ increases. Taking into account that Reδ is always equal to 4.4817, Equation 9 can be rewritten as:

$$f = 8 \left[ \frac{(\delta + 1)(\delta + 2)}{2^{\frac{3}{2}} \cdot 2.1763} \right]^{-\frac{1}{2}} \left[ \frac{0.537}{\delta^{1.131}} \right]^{\frac{1}{2}}.$$  

(10)

Taking into account that the experimental range of $\delta$ (0.162–0.209) is narrow, a constant value equal to the mean experimental value 0.1787 is used. Therefore, Equation 10 can be rewritten as:

$$f = 4.02 \frac{5^{0.91}}{F^{1.187}}.$$  

(11)

Equation 11 states that flow resistance increases with slope that surrogates the effect of bed-load transport. Furthermore, Equation 11 establishes that the Darcy–Weisbach friction factor $f$ increases with a power of slope gradient characterized by an exponent of the slope approximately equal to one. This result suggests that a slope quasi-independence hypothesis of rill velocity in eroding rills may be correct (Govers, 1992). Thereby, the applicability of Equation 1 is linked to the ability of the investigated rills to adjust their geometry (wetted perimeter and roughness), which affects the hydraulic roughness and hence the friction factor values.

According to Hessel et al. (2003), this result can be also justified by an energy-based point of view. Because the rill erosion process is more active when the slope increases, then a greater portion of flow energy is used for particle erosion from the rill boundary and sediment transport within the rill. If more energy is used for erosion-transport phenomena on a steep slope than on a gentle one, then less energy will be available for increasing velocity. Erosion and sediment transport phenomena interact with (depend and affect) flow characteristics driving the rill geometry adjustment, the eroded particle motion, and the flow velocity. The available energy, which is dependent on plot slope, is used for driving the water flow and its kinematic characteristics which in turn affect erosion and transport within the rill.

Taking into account that the experiments presented in this investigation were carried out on a clay soil, the obtained results allowed to expand the range of applicability of the conclusion by Takken et al. (1998). In other words, the experiments confirmed the hypothesis that flow velocity in rills should be independent of slope even if the rill is
incised on a clay soil. Experiments should be carried out for further testing the slope-independence hypothesis of rill velocity using soils with different texture composition.

4 | CONCLUSIONS

The problem of determining the average velocity in a rill is still open because the available experimental data suggest that the rill flow cannot be well described by the Manning’s or Darcy–Weisbach equation. Furthermore, the interaction between bed morphology, flow conditions, and flow erosivity may require modification of the classic flow resistance formulations.

Many previous scientists have noticed that rill flow velocities tend to be independent of slope, and therefore, a model that uses Darcy–Weisbach’s equation to predict rill flow hydraulics must take into account the slope-independence of flow velocity by allowing the Darcy–Weisbach friction factor to increase with slope gradient.

In this paper, a new flow resistance equation, theoretically deduced by dimensional analysis and self-similarity theory, was tested by carrying out experimental runs on different sloping plots in which mobile bed rills were incised. The theoretical expression of the Darcy–Weisbach coefficient, which is based on the hypothesis that a power flow velocity-distribution can be applied, contains the two parameters of the velocity profile.

The relationship between the velocity profile parameter $\Gamma$, the channel slope, and the flow Froude number, which was calibrated by the previously collected 106 rill reach measurements, was tested using measurements carried out on plots having slopes equal to 22% and 9%. The measurements carried out in these latter slope conditions confirmed that the Darcy–Weisbach friction factor can be accurately estimated by the proposed theoretical approach (with estimate errors that were less than or equal to $\pm$20% for 95.6% of cases). Finally, the theoretical expression of the Darcy–Weisbach friction factor (Equation 5) coupled with Equation 8 confirmed that the slope independence hypothesis of rill velocity stated by Govers (1992) is supported by these data.

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REFERENCES


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