Cost function optimization for predictive control of a five-phase IM drive

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Summary
Model predictive control has been used, for some time now, as a method to directly control power converters in electrical systems. The usual practice is tuning the cost function of the controller to obtain a certain compromise solution over the whole operating range of the system. This method is extended here to consider multiple, locally optimal, and tunings. The design objectives (tracking error, switching frequency, etc) are used to define a unique performance index that is locally optimized. In this way, the parameters of the cost function are linked to the current operating point. The tuning at each operating point is obtained numerically solving the optimization of the performance index. Although the idea can be applied to induction machines with any number of phases, in this paper, a five-phase induction motor is considered for presentation. This system is a demanding case due to the extra number of phases compared with the traditional three-phase motor. Simulation and experimental results are presented to assess the proposed predictive controller.

KEYWORDS
cost functions, induction machines, optimization, predictive control

1 | INTRODUCTION

Model predictive control (MPC)\textsuperscript{1} has been used in electrical systems such as power converters,\textsuperscript{2} induction machines (IMs),\textsuperscript{3} permanent magnet machines,\textsuperscript{4} grid control,\textsuperscript{5} etc. In most cases the MPC uses a modulation stage to interface with the power converter that has a finite number of configurations. This is typically done by using pulse width modulation (PWM) or related techniques to drive a voltage source inverter (VSI).\textsuperscript{6}

Model predictive control has also been used as a versatile control technique for systems using power converters avoiding the use of modulation stages such as PWM modules.\textsuperscript{7} The elimination of the modulation stage (ie, the production of a mean voltage by means of PWM) brings a fast response which is quite useful to cope with reference tracking and perturbation rejection problems. In addition, the flexibility of MPC to consider different control criteria and constraints for different systems has spurred a number of applications. In particular, in the case of IM drives, MPC strategies for current, torque, and speed tracking have been successfully implemented.

Abbreviations: AC, alternating current; DC, direct current; IM, induction machine; MPC, model predictive control; PCC, predictive current control; RMS, root mean squared; PWM, pulse width modulation; SC, switch changes; THD, total harmonic distortion; VSI, voltage source inverter.
Multiphase IMs are special cases of AC machines with more than three-phases. Their inherent traits have made them receive attention in both academia and industry. They are preferred in some applications due to their higher power density, increased robustness, and lower harmonic content. In the realm of multiphase drives, finite control set MPC is an emerging technique, where the particular case of predictive current control (PCC) considered here is one of the most popular. It easily allows to treat the extra number of phases considering different electromechanical issues relevant to control applications such as, copper losses, VSI losses, switching frequency, and torque ripple. In particular, the PCC scheme has been used with the five-phase IM.

The design of PCC faces a trade-off between conflicting criteria. For instance, accurate tracking in \( \alpha - \beta \) subspace is often found to produce current ripple in the \( x - y \) plane. As a result, stator copper losses and current harmonics might be increased. Since \( \alpha - \beta \) tracking error is related to speed ripple, this means that the trade-off is extended to the mechanical part of the drive. Other interesting variables share also some form of trade-off as shown in the work of Arahal et al. The usual practice is to tune the PCC to obtain a certain global compromise solution, as done in the work of Lim et al. This practice is tedious, cumbersome, and not too intuitive as the weighting factors influence the results in a highly nonlinear way (see the work of Arahal et al). To tackle this problem, a number of proposals have appeared recently in the electrical systems literature. The new methods include neuronal approximations, elimination of weighting factors, fuzzy decision making, and other techniques (see the work of Mamdouh et al for a review). In the more general literature concerned with automatic control, different approaches can be found such as on-line tuning.

In this paper, the design objectives (in terms of some figures of merit such as losses, tracking quality, and so on) are used to define a unique performance index that quantifies the goodness of a certain tuning. The quantification is done according to some variables with physical meaning and to the preferences of the designer. The index is then used as a guide to derive a cost function tuning that is optimal for a particular operating point. By repeating the procedure for other operating points, different parameters of the PCC are considered, producing an scheduled controller. In this way, the trade-offs can be considered locally, producing potentially better results.

Although the proposed method can be applied to any IM drive, a five-phase IM is considered for the sake of concreteness. Please note that the five-phase IM constitutes a demanding application example. In the next section, the MPC control scheme for a five-phase IM is revisited. The proposed cost function tuning method for the scheduled controller is presented in Section 3, where the performance index and the operating space discretization are introduced. Simulation results and experimental results are presented in Sections 4 and 6, respectively, followed by the conclusions.

2 | PREDICTIVE CURRENT CONTROL OF THE FIVE-PHASE IM

Figure 1 presents the diagram of PCC of a five-phase, VSI-driven IM. At discrete time \( k \), the controller computes the optimal state of the VSI for the next sampling period \( u(k+1) \). The VSI, in turn, supplies a voltage \( v(k+1) \) whose objective is the generation in the stator of currents \( i_s(k) \) that follows a reference trajectory \( i^*(k) \). These trajectories are in most cases sinusoidal waveforms whose amplitude \( I^* \) and frequency \( f_e \) are set by an external control loop responsible for speed/position tracking of the drive.
The PCC strategy uses exhaustive optimization over all possible control moves and selects the one that minimizes the cost function at \( k + 2 \). The receding horizon strategy is then used, issuing the actual control action and repeating the procedure at the next sampling period. Please recall that, in PCC, the time needed to compute the control action is close to the sampling period; thus, a whole sampling time delay is introduced as explained in the work of Lim et al.\(^9\)

Please notice that this scheme uses a control horizon of just one and a prediction horizon of two. This is due to the fact that very short sampling periods are used (ranging from 40 to 100 microseconds). Nevertheless, the scheme is model-based, includes an optimization phase, and uses the receding horizon idea, which are basic constituents of any MPC technique.\(^1\)

A dynamic model of the IM and a static model of the VSI are used to derive the two-step ahead predictions for each possible control action (VSI state). The IM model is computed from the standard IM equations in phase variables. For an \( n \)-phase IM, the vector space decomposition technique maps the \( n \)-phase space into plane \( \alpha - \beta \) (involved in the energy conversion) and \((x - y)^1\) to \((x - y)^{(n-4)/2}\) subspaces that are related to the losses and represents supply harmonics of the order \( 10n \pm 3 \). Zero sequence harmonics (5n with \( n = 1, 2, 3, ... \)) are not considered because the neutral point is isolated. For the particular case of the five-phase IM, the VSI must have five sets of switches; then, the voltage vectors take the distribution shown for the \( \alpha - \beta \) and \( x - y \) subspaces in Figure 2.

In order to derive a prediction model, the stator current components in \( \alpha - \beta \) and \( x - y \) are used as state variables together with the \( \alpha - \beta \) rotor currents. In this way, \( x(t) = (i_{s\alpha}, i_{s\beta}, i_{sx}, i_{sy}, i_{r\alpha}, i_{r\beta})^\top \) is taken as state vector for the modelling. The drive equations are

\[
\dot{x}(t) = A_c(\omega_r(t))x(t) + B_c v(t) \\
y(t) = Cx(t).
\]

The input signal in this model is a vector of stator voltages \( v(t) = (v_{s\alpha}, v_{s\beta}, v_{sx}, v_{sy})^\top \); the output signal is a vector of stator currents \( y(t) = (i_{s\alpha}, i_{s\beta}, i_{sx}, i_{sy})^\top \). Matrices \( A_c \) and \( B_c \) depend on the rotor electric speed \( \omega_r(t) \) and the following machine parameters: stator and rotor resistances \( R_s \) and \( R_r \), stator and rotor inductances \( L_s \) and \( L_r \), stator leakage inductance \( L_{ls} \), and mutual inductance \( L_M \)

\[
A_c(\omega_r(t)) = \begin{pmatrix}
-a_{s2} & a_{m4} & 0 & 0 & a_{r4} & a_{l4} \\
-a_{m4} & -a_{s2} & 0 & 0 & -a_{r4} & a_{l4} \\
0 & 0 & -a_{s3} & 0 & 0 & 0 \\
0 & 0 & 0 & -a_{c3} & 0 & 0 \\
a_{s4} & -a_{m5} & 0 & 0 & -a_{r5} & -a_{l5} \\
a_{m5} & a_{s4} & 0 & 0 & a_{l5} & -a_{r5}
\end{pmatrix}, \quad B_c = \begin{pmatrix}
c_2 & 0 & 0 & 0 \\
0 & c_2 & 0 & 0 \\
0 & 0 & c_3 & 0 \\
0 & 0 & 0 & c_4 \\
-4 & 0 & 0 & 0 \\
0 & -c_4 & 0 & 0
\end{pmatrix}
\]

The aforementioned coefficients are \( c_1 = L_sL_r - L_M^2 \), \( c_2 = L_r/c_1 \), \( c_3 = 1/L_{ls} \), \( c_4 = L_M/c_1 \), \( c_5 = L_s/c_1 \), \( a_{s2} = R_c2 \), \( a_{s3} = R_c3 \), \( a_{s4} = R_c4 \), \( a_{r4} = R_r4 \), \( a_{r5} = R_r5 \), \( a_{l4} = L_c4\omega_r \), \( a_{l5} = L_c5\omega_r \), \( a_{m4} = L_Mc4\omega_r \), and \( a_{m5} = L_Mc5\omega_r \).

The VSI converts gating signals into stator voltages according to \( v(t) = V_{dc}TMu(t) \), where \( V_{dc} \) is the DC-link voltage, \( T \) is the connectivity matrix, and \( M \) is a coordinate transformation matrix depending on the spatial distribution of the IM
The parameters for the PCC will be obtained optimizing a performance index that incorporates the design objectives. The windings
\[
T = \frac{1}{5} \begin{bmatrix}
4 & -1 & -1 & -1 & -1 \\
-1 & 4 & -1 & -1 & -1 \\
-1 & -1 & 4 & -1 & -1 \\
-1 & -1 & -1 & 4 & -1 \\
-1 & -1 & -1 & -1 & 4
\end{bmatrix} , \quad
M = \frac{2}{5} \begin{bmatrix}
1 & \cos \theta & \cos 2\theta & \cos 3\theta & \cos 4\theta \\
0 & \sin \theta & \sin 2\theta & \sin 3\theta & \sin 4\theta \\
1 & \cos 2\theta & \cos 4\theta & \cos \theta & \cos 3\theta \\
0 & \sin 2\theta & \sin 4\theta & \sin \theta & \sin 3\theta \\
1/2 & 1/2 & 1/2 & 1/2 & 1/2
\end{bmatrix}
\]

where matrix \(A\) is not constant but depending on the electrical frequency \(A(\omega_r(k))\). The measurement vector \(i_s(k) = (i_a, i_b, i_c, i_d)\) is obtained from measurements of stator currents \(i_s\) for phases \(a - b - c - d - e\) transformed into \(a - \beta\) and \(x - y\) subspaces. Vector \(G(k)\) accounts for the dynamics due to rotor currents that are usually not measured. It constitutes a term that must be estimated at each \(k\); for this task, several options exists in the work of Martín et al.12 Finally, in Equation (5), the control signal \(u(k)\) is a vector of gate signals of the VSI legs \(u(k) = (K_1, K_2, \cdots, K_5)^T\), where \(K_i \in \{0, 1\}\) for \(i = 1, \ldots, 5\). Each phase of the motor can be either connected to the positive (\(K_i = 1\)) or negative (\(K_i = 0\)) rail of the DC-link, producing \(2^{5}\) different possible values for \(u\), out of which just 31 different voltages are produced, as shown in Figure 2).

Joining the IM and VSI models, and after appropriate discretization, the prediction model can be obtained as
\[
\hat{i}_s(k + 2|k) = A(\omega_r(k))i_s(k) + B_1u(k) + B_2u(k + 1) + G(k),
\]

where
\[
\hat{i}_s(k + 2|k) = (\hat{i}_a(k + 2), \hat{i}_b(k + 2), \hat{i}_c(k + 2), \hat{i}_d(k + 2), \hat{i}_e(k + 2))
\]

This quantity is of importance as the switching frequency is \(f_{sw} = SC/T_s\), where \(T_s\) is the sampling time of the controller. This frequency must be kept within limits for safe operation of the VSI and also to keep commutations losses below some limit. With these considerations, the cost function that will be used is
\[
J = ||\hat{e}_x||^2 + \lambda_1 ||\hat{e}_y||^2 + \lambda_2 SC,
\]

where \(||.||\) denotes vector modulus, \(\lambda_1\) is a parameter to give more importance to \(x - y\) penalization, and \(\lambda_2\) is a similar parameter that allows to give more importance to switch changes penalization.

This configuration has been used in multiphase electrical systems where two or more subspaces appear. The higher number of phases (compared to the standard three-phase case) provides further room for optimization, and more tuning possibilities at the cost of higher computational cost compared with the three-phase case.

Please note that the discrete time and quantized actuation nature of this control scheme makes it a difficult object for stability analysis. The conventional approaches18 do not apply, and in fact, this topic is an open problem. Two lines of research have been proposed; the first one is based on Lyapunov stability theory;19 the second one considers hybrid systems where a set of discrete states interacts with a continuous dynamic system.19 Since the formal stability of the system cannot be readily demonstrated, this issue is rarely discussed in the literature. Some results, however, have been derived for specific systems mainly using quadratic cost functions. The interested reader can consult the works of Aguilera and Quevedo20 and Bordons and Montero.21

### 3 Cost Function Parameter Selection

The parameters for the PCC will be obtained optimizing a performance index that incorporates the design objectives. The most commonly found objectives are reference tracking in \(a - \beta\) and \(x - y\) subspaces; additionally, other quantities such as total harmonic distortion (THD), flux ripple, speed ripple, and switching frequency are considered.

Tracking error of stator currents in \(a - \beta\) subspace is directly related to current THD since the reference current is sinusoidal. Moreover, speed ripple is a low-pass filtered version of the torque ripple (mechanical inertia), which, in turn,
directly depends on the ripple of the $\alpha - \beta$ currents. Tracking in the $x - y$ plane affects the stator copper losses and THD. Consequently, keeping $x - y$ currents close to zero simultaneously improves the power quality and drive efficiency.

A consequence of the discrete-time direct control of power converters (not using modulation stages) is that the switching frequency is not constant. This means that the number of switching changes at the VSI can differ from one electrical cycle to another. The reason is that the PCC chooses at each sampling period the most adequate control action, including the possibility of issuing the same action as in the previous sampling period producing zero commutations. The average switching frequency for an electrical cycle can be considered to deal with the problem of thermal losses at the VSI. The semiconductors that form the VSI cannot be subject to arbitrarily large switching frequencies because the heat would reduce their life span. Safety limits are thus placed on operation in terms of sustained switching regimes. These limits prevent the VSI from overheating due to commutation losses. In addition, average switching frequencies are usually considered for hardware selection (e.g., standard insulated-gate bipolar transistors or silicon carbide–based power switches) and for VSI losses estimation.

In this paper, root mean squared values of tracking error for $\alpha - \beta$ and $x - y$ are considered as design objectives together with the average sampling frequency. For the sake of readability, the following definitions are introduced:

\[
E_{\alpha\beta} = \frac{1}{2} \left( e_{\alpha}^{\text{RMS}} + e_{\beta}^{\text{RMS}} \right).
\]

\[
E_{xy} = \frac{1}{2} \left( e_{x}^{\text{RMS}} + e_{y}^{\text{RMS}} \right).
\]

\[
E_{sw} = \frac{1}{T_s (k_2 - k_1 + 1)} \sum_{k=k_1}^{k_2} SC(k),
\]

where indices $k_1$ and $k_2$ are selected to span a whole electric cycle; thus, $T_s (k_2 - k_1 + 1) = 1/f_e$. These indices are related to the actual tracking error in $\alpha - \beta$ plane, the regulation error in $x - y$ plane, and the average switching frequency. Taking this into account, the following performance index is proposed:

\[
\Pi = DZ(E_{\alpha\beta}) + DZ(E_{xy}) + DZ(E_{sw})
\]

where DZ are dead-zone like functions defined as

\[
DZ(x) = \begin{cases} 
0, & x \leq D_x \\
\frac{x}{U_x}, & x > D_x.
\end{cases}
\]

where threshold values $D_x$ and upper limits $U_x$ are defined separately for each term in Equation 11 by considering $x = E_j$ for $j \in \{\alpha\beta, xy, sw\}$.

This choice for the performance index allows to take into account in an easy manner different criteria according to the particular application. Clearly, the threshold values $D_{\alpha\beta}$, $D_{xy}$ and $D_{sw}$ represent the points at which every term begins to be considered significant. Any quantity below its threshold is treated as zero. The upper limits $U_{\alpha\beta}$, $U_{xy}$ and $U_{sw}$ in turn allow to weight variables of different magnitudes by providing a point where each one is considered equally important.

For instance, if commutation losses are important, as in high power applications, then $D_{sw}$ and $U_{sw}$ should be lowered. If current quality is of concern, then $D_{\alpha\beta}$ and $U_{\alpha\beta}$ must take lower values, etc. In the simulations and experimental results, the values used are those indicated in Table 1. This choice is just an example in which it has been considered that $x - y$ currents below $D_{xy}$ and switching frequencies below $D_{sw}$ are not important, whereas tracking minimization is sought through setting $D_{\alpha\beta} = 0$.

### 3.1 Operating space discretization

In order to optimize the performance index $\Pi$, a suitable discretization of the operating space is first made. Expert knowledge suggests using speed and load of the IM as indicators of the operating point. This knowledge comes from the fact...
that PCC applied to IM produces results that are sensitive to speed and load as shown in the work of Lim et al.\textsuperscript{9} This hypothesis will later be confirmed by the results. Then, the discretization can be made by considering different values of speed $\omega$ and stator current reference amplitude $I^\ast$. The space of operating points $\Phi$ is sampled using a Cartesian grid $\Phi_m = S_\omega \times S_I$, with $S_\omega = \{\omega_1, \omega_2, \ldots, \omega_N_\omega\}$ and $S_I = \{I_1, I_2, \ldots, I_N_I\}$. Any operating point in this set can be defined from a pair of indices $(k,j)$ as $\phi_{k,j} = (\omega_k, I_j)$. Please note that the usual technique of PCC is equivalent to considering $N_\omega = 1, N_I = 1$. Moreover, it is worth remarking that, for higher values of $N_\omega$ and $N_I$ a finer discretization is made, potentially bringing better results at the cost of higher computing load.

The optimization of cost functions is made for each operating point $\phi_{k,j}$ by solving the following problem:

$$\begin{align*}
\left( \lambda_1^{o}(k, j), \lambda_2^{o}(k, j) \right) = \underset{(\lambda_1, \lambda_2)}{\text{argmin}} \Pi(\lambda_1, \lambda_2, \omega_k, I_j).
\end{align*}$$

This problem is a general nonlinear optimization problem with nonnegative continuous variables $(\lambda_1, \lambda_2)$. The solution can be found using a variety of algorithms that are suited for a general optimization task. Note that the problem is not convex (ie, local minima can exist). In this paper, a Newton-Raphson algorithm is used with a posterior phase that uses Monte Carlo to corroborate the results. The discretization of the operating space has been made using seven divisions for $\omega$ and 17 divisions for $I^\ast$ evenly distributed on their ranges. This discretization takes into account that larger variations are observed in the $I^\ast$ axis than in $\omega$ axis. Moreover, the number of divisions produces an optimization problem that is manageable in terms of computing time with any modern general-purpose computer.

Figure 3 shows the obtained optimal values of $\lambda_1$ and $\lambda_2$ according to Equation 13 and using a color code. The vertical axis corresponds to $\omega$ and the horizontal axis to $I^\ast$ using a per unit system. A bicubic interpolation has been used to fill the areas surrounding the grid points. Large variations in both parameters ($\lambda_1, \lambda_2$) can be seen across the operating space. For $\lambda_1$, the values found are in the interval $[0.06, 10]$, and for $\lambda_2$, the range extends to $[0.01, 236] \cdot 10^{-5}$.

**FIGURE 3** Optimal values for $\lambda_1$ (left) and $\lambda_2$ (right) for operating points in $\Phi$ [Colour figure can be viewed at wileyonlinelibrary.com]

4 | SIMULATION RESULTS

Next, the simulation results for a particular five-phase IM are presented. The IM has the following characteristics: stator resistance 19.4 ($\Omega$), rotor resistance 6.8 ($\Omega$), stator inductance 101 (mH), rotor inductance 39 (mH), mutual inductance 657 (mH), nominal speed 1000 (rpm), power 1 (kW), and pole pairs 3.

The IM has been simulated using a Runge-Kutta algorithm in which the controller is a discrete-time subsystem with a sampling time of 80 ($\mu$s). This value is within the range usually found for PCC and can be obtained by a variety of modern digital signal processors.

The optimization of the performance index has been performed using a general minimization algorithm with two steps, a Newton-Raphson search, followed by a Monte Carlo phase to corroborate results. The discretization of the operating space has been made using seven divisions for $\omega$ and 17 divisions for $I^\ast$ evenly distributed on their ranges. This discretization takes into account that larger variations are observed in the $I^\ast$ axis than in $\omega$ axis. Moreover, the number of divisions produces an optimization problem that is manageable in terms of computing time with any modern general-purpose computer.

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It is worth comparing the results obtained with the traditional method (fixed parameters) and with the proposed one (scheduled parameters) in terms of the performance index $\Pi$ and also in terms of the different figures of merit $E$. Two cases of the traditional PCC with fixed parameters are presented for comparison purposes. The first one (case A) uses a commonly found pair of values $\lambda_1 = 0.5$, $\lambda_2 = 0$. This case seeks $\alpha - \beta$ tracking allowing for a certain amount of $x - y$ current to appear and does not penalize VSI switch changes, so commutation losses can be significant (see the work of Arahal et al.13). The second case (case B) is obtained by minimizing $\Pi$. This case represents the best tuning for the traditional approach in terms of the performance index and is attained for $\lambda_1 = 1$, $\lambda_2 = 9.2842 \cdot 10^{-4}$. It also can be seen as an extreme case where $N_\omega = N_I = 1$. Both cases are thus significative for comparison purposes.

Figure 4 shows the optimal values for $\Pi$ using the scheduled locally optimal cost functions (left) and for the two traditional cases. It shows relatively low values except for operating points close to the limit of the IM (high speeds and loads). For the traditional PCC, the performance index degrades quicker and is, in general, higher. The lowest portion of the color palette has been intentionally filled black to better appreciate the superiority of the proposed scheme.

In order to further test the advantages of the proposed scheme, Table 2 presents the mean $\mu$, the standard deviation $\sigma$, the minimum and maximum values of $E_{\alpha\beta}$, $E_{xy}$, $E_{sw}$, and $\Pi$. It can be seen that the proposed technique does indeed provide lesser values with less variation.

It is worth remarking that some trade-offs exist in the operation of the IM. This is apparent in some cases present in Table 2. Comparing cases A and B of the traditional method, it can be seen that a reduction in some measures. For instance, $E_{sw}$ is lower in case B with respect to case A, but this decrease is accompanied by an increase in $E_{\alpha\beta}$. These changes do not take place globally; some operating points are more prone to high values of $E_{sw}$, while others present a deterioration of $E_{\alpha\beta}$. The virtue of the proposed scheme is that it allows the designer to locally shape the obtained behavior.

<table>
<thead>
<tr>
<th>Proposed (optimal ($\lambda_1$, $\lambda_2$))</th>
<th>min</th>
<th>max</th>
<th>$\mu$</th>
<th>$\sigma$</th>
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<tr>
<td>$E_{\alpha\beta}$ (A)</td>
<td>0.020024</td>
<td>0.063644</td>
<td>0.032692</td>
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<tr>
<td>$E_{xy}$ (A)</td>
<td>0.022910</td>
<td>0.294608</td>
<td>0.036921</td>
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<td>$E_{sw}$ (kHz)</td>
<td>1.064257</td>
<td>5.126836</td>
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<tr>
<td>$\Pi$</td>
<td>1.614637</td>
<td>10.49649</td>
<td>2.215974</td>
<td>1.071710</td>
</tr>
<tr>
<td>Traditional (case A)</td>
<td>min</td>
<td>max</td>
<td>$\mu$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>$E_{\alpha\beta}$ (A)</td>
<td>0.023935</td>
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<td>0.031125</td>
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<tr>
<td>$E_{xy}$ (A)</td>
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<td>0.174001</td>
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<tr>
<td>$E_{sw}$ (kHz)</td>
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<td>1.206346</td>
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<tr>
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</tr>
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<td>$\sigma$</td>
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<tr>
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EXPERIMENTAL RESULTS

In order to test the feasibility of the scheduled PCC proposed here, a number of tests have been carried out using a laboratory set up. The test rig includes (see Figure 5) a 30-slot symmetrical five-phase IM (with nominal parameters already presented in Section 4), a DC power supply, which sets the DC-link to 300 V, and a pair of three-phase SKS21F Semikron inverters. One of the inverter has an open leg as only five phases are needed. The connection of the different elements is shown in Figure 5. Variable load conditions can be applied to the IM in a controlled manner using an independent DC machine mechanically coupled to the five-phase IM.

The control program runs in a TM320F28335 Texas instrument digital signal process using a MSK28335 Technosoft board for interfacing with current and speed sensors. The same conditions are considered for all controllers.

The first set of tests (Table 3) compares the results of using the optimal parameters versus non-optimal ones for a particular operating point. The operating point is close to the nominal speed and load of the IM and is characterized by \( \omega = 500 \) (rpm), \( I^* = 2 \) (A). The optimal parameter for this operating point is obtained for \( \lambda_1 = 0.5 \). Other choices of the cost function parameter are included for comparison, all of which are usually found in the literature. Comparing the results of the optimal choice with the first entry of Table 2 gets a lower value of \( E_{a\beta} \), a much lower value of \( E_{xy} \) and a moderate increase of \( E_{sw} \). Comparing with the last entry of the table, the optimal parameters provide a lower value of \( E_{a\beta} \), a lower value of \( E_{sw} \) and a small increase of \( E_{xy} \).

The second set of tests (Table 4) shows the variation of some figures of merit when the operating point is changed but using the same control parameter. It can be seen that the different indices take different values that can be far from the optimal.

Finally, the trajectories of stator currents in \( \alpha \) and \( x \) axis are shown in Figure 6. For the tests, the reference value \( i^*_\alpha \) for the \( \alpha \) component is a sinusoidal trajectory whose amplitude is set by the outer control loop in charge of speed regulation. In addition, the reference value \( i^*_x \) is zero (as in most cases) seeking the minimization of losses in \( x - y \) subspace. It can

![FIGURE 5 Experimental test rig](Colour figure can be viewed at wileyonlinelibrary.com)

<table>
<thead>
<tr>
<th>Regime</th>
<th>Parameters</th>
<th>( E_{a\beta} ) (A)</th>
<th>( E_{xy} ) (A)</th>
<th>( E_{sw} ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega = 500 ) (rpm), ( I^* = 2 ) (A)</td>
<td>( \lambda_1 = 0.1 )</td>
<td>0.077934</td>
<td>0.125918</td>
<td>1237</td>
</tr>
<tr>
<td>( \omega = 500 ) (rpm), ( I^* = 2 ) (A)</td>
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<td>0.076371</td>
<td>1297</td>
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<td>0.075024</td>
<td>1377</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Regime</th>
<th>Parameters</th>
<th>( E_{a\beta} ) (A)</th>
<th>( E_{xy} ) (A)</th>
<th>( E_{sw} ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.108028</td>
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<tr>
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<td>( \lambda_1 = 0.1 )</td>
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<td>0.125918</td>
<td>1237</td>
</tr>
<tr>
<td>( \omega = 600 ) (rpm), ( I^* = 1.6 ) (A)</td>
<td>( \lambda_1 = 0.1 )</td>
<td>0.078211</td>
<td>0.114006</td>
<td>1400</td>
</tr>
<tr>
<td>( \omega = 700 ) (rpm), ( I^* = 1.6 ) (A)</td>
<td>( \lambda_1 = 0.1 )</td>
<td>0.075604</td>
<td>0.118167</td>
<td>1071</td>
</tr>
</tbody>
</table>
be seen that a lower content of $x-y$ currents is found when using the optimal parameters. Again, it must be stressed that the proposed method allows to give more relevance to one particular criteria over others.

6 | CONCLUSIONS

The use of scheduled parameters for PCC in a multiphase IM has been proposed. Compared with previous approaches for cost function tuning, this work provides a means to consider the existing trade-offs locally, allowing a more adequate treatment for each operating point. In addition, optimality for the operating points included in the partition of the operating space can be achieved thanks to the use of a unique performance index.

The design of the proposed performance index includes different criteria normally used in the realm of drive control. Dead-zone like functions have been proposed to help translating operational objectives into mathematical form. In this way, the performance index allows the designer to transform a nonintuitive problem of selecting the weighting factors of the cost function to a problem where six parameters with physical meaning must be defined. From these six parameters, the weighting factors are derived off-line solving an optimization problem. The design is thus made on a local basis, by optimizing the performance index with respect to the parameters of the cost function. The optimization problem has an affordable computing burden for modern computers. Although the proposal is applicable to different types of electrical systems, a five-phase IM has been used as a case example to show the viability of the proposed scheme. In fact, for an $n$-phase IM, other $x_i-y_i$ planes can appear that are easily considered including another term in the cost function. For multilevel systems, the extension of the approach is even simpler as only the allowed VSI states need to be updated. Finally, considering other criteria for PCC tuning requires just adding or removing some $E$ terms from the definition of $\Pi$.

The results show that the technique has the potential of shaping the obtained response according to different criteria, namely, tracking, copper losses, and commutation losses. The experimental results show that improved performance indices can be achieved compared with the traditional approach.

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