RESEARCH ARTICLE

Outage performance of relay selection with spatially random relays using RF energy harvesting

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ABSTRACT

To enable green wireless networks, one appealing approach is to deploy energy harvesting (EH) relays to assist the source transmission. Unlike conventional relays relying on fixed power supplies, EH relays make use of the energy collected from the RF radiation of the source node, and thus, they do not introduce extra energy cost to the network. This paper presents an analytical study to assess the efficacy of EH relays when the one with the maximal end-to-end signal-to-noise ratio is selected to perform data relaying while others perform EH. Because the action (either harvesting energy or forwarding data) of one EH relay affects those of others, exact performance analysis is not tractable. Additionally, relay density and positions may be random, which further complicates the analysis. Our analysis is conducted based on the hypothesis that each EH relay has an equal chance to be selected. This hypothesis allows for analytical tractability and is of importance to EH relays because otherwise some may drain their batteries fast. We identify the conditions under which the aforementioned hypothesis is valid. Our analysis also considers two variants of amplify-and-forward relays with and without using channel state information. Numerical results are presented to validate the analysis accuracy along with extensive discussions on the impact of numerous system parameters. Copyright © 2015 John Wiley & Sons, Ltd.

KEYWORDS
energy harvesting; cooperative relaying; amplify-and-forward; Poisson point process; performance analysis

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1. INTRODUCTION

Energy harvesting (EH) technique has been envisioned as an attractive solution to facilitate self-sustaining, cost-friendly, and energy-efficient wireless networks. Thanks to the recent advancement in highly efficient circuits and rectifying antennas [1], an EH node is capable of deriving energy from external energy sources (such as solar, wind, kinetic, and ambient radio frequency (RF) energy). Several recent projects have demonstrated the potential of employing EH wireless nodes in performing sensing and data transmission [2,3].

Among various EH techniques, wireless charging through RF radiation is of particular interest. Also known as wireless power transfer, replenishing energy from ambient RF radiation can be realized in two different approaches. In [4], the authors propose to wirelessly charge mobile stations by installing some fixed power stations randomly distributed in the network. Theoretical analysis shows that such a strategy can satisfy the outage constraint provided with sufficient density of the deployed power stations. Another approach, referred to as simultaneous wireless information and power transfer, reuses the signal source, such as the WiFi access point, cellular base station, and microwave towers, to perform wireless power transfer, and thus, no extra infrastructure and operational costs are added to existing networks.

Based on simultaneous wireless information and power transfer, some recent work studies the use of EH relays to enable reliable and green wireless networks, where EH relays are deployed to assist the information delivery from the source to the destination node. Unlike conventional relays that are equipped with fixed power supplies, EH relays eliminate the need of power chords, and thus, they are self-sustainable and can be conveniently installed. The major challenge to EH relays is threefold. Firstly, the energy neutrality constraint, that is, the available energy to transmit, must be less than the harvested energy, which depends on the energy profile of the energy source [5] and the energy conversion efficiency of the energy harvester [1]. To overcome the mismatch between the charging and discharging profiles, efficient power management and power allocation schemes are of vital importance [6,7]. As to the energy conversion efficiency, it is closely related
to the nature of energy sources and harvesting methods. For example, photovoltaic harvesting may achieve efficiencies ranging from 1–20%, depending on the materials of solar cells [8]. Secondly, the energy causality, that is, the energy harvested in the future, can not be used for current transmission [9]. A common practice is thus to store the harvested energy in an energy buffer (e.g., a rechargeable battery or a super capacitor) for later use. Still, an EH relay may not always be available because the energy harvested is often much smaller than that required for transmission, and thus, relays may lack enough energy to transmit. Thirdly, the energy half-duplex constraint, that is, the energy buffer, can not be charged and discharged at the same time [10]. Consequently, a relay performing EH will opt out from forwarding data.

To assess the efficacy of EH relays, theoretical analysis on the performance of cooperative networks based on EH relays is critically valuable and has been carried out in some recent work. For instance, the asymptotic analysis conducted in [5] indicates that cooperative relaying based on EH relays can fully exploit diversity only if relays are energy unconstrained, that is, the rate that the relay harvests energy exceeds the average rate of energy consumption. In their study, relays are assumed to harvest energy from an external energy source, and the energy storage at relays is of infinite capacity. In [11–13], the RF radiations from the source signal are used as the energy source for EH relays. In this context, relays need to switch between the EH mode or data forwarding mode according to the energy half-duplex constraint. Considering a simplified network with a single relay, Krikidis et al. [11] study the optimal mode switching problem for minimizing the outage probability subject to the finite energy storage at the relay node. The same scenario is considered in [12], where the focus is on utilizing relay feedback to improve the channel utilization. On the other hand, Nasir et al.[13] consider battery-free relays such that the harvested energy needs to be used immediately. The optimal power splitting ratio between the power consumed for signal processing and that for transmission is obtained numerically.

The aforementioned theoretical studies [5,10,11,13] either ignore the exponentially decayed propagation loss with distance or assume constant distances between nodes. In practice, relays may be randomly distributed in the network, and their positions also change with time. Spatial randomness of the relay locations is taken into account in [14], and its impact to the outage performance of EH relays with power splitting is analyzed based on stochastic geometry. It is shown that using EH relays achieves the same diversity gain as the case with conventional relays for most practical values of path loss exponent. Stochastic geometry has also been used to analyze the performance of point-to-point transmission between EH nodes. For example, Flint et al. [15] derive the power and transmission outage probability of a sensor node, which harvests energy from multiple RF energy sources randomly located in the proximity.

This paper focuses on the performance analysis of relay selection with EH relays that harvest energy using the RF signal transmitted from the source node. The major contributions of this paper are highlighted as follows.

- The paper proposes an analytical framework, taking into account two important constraints of spatially random EH relays, namely, the finite energy storage and the half-duplex hardware. Such a joint consideration is important to evaluate the practical values of EH relays, but exact analysis is not tractable because data relaying and EH are two coupled processes. Such a scenario has been considered in [11] and [12], which model the evolution of battery status via a Markov chain. However, their works ignore the impact of path loss and assumes a single relay in the network. Our work investigates a more practical network setting with multiple relays randomly distributed in the network. We identify in which conditions that a one-dimensional discrete-time Markov chain (DTMC) model can be useful to capture the change of the relay battery status with time.
- Our analysis considers both channel state information (CSI)-based and blind amplify-and-forward (AF) relays. For both cases, we derive the outage probability when the relay that offers the maximal end-to-end (e2e) signal-to-noise ratio (SNR) is selected to forward the source information, while the remaining relays harvest energy from the source signal. The SNR maximization selection rule has been widely studied in the literature for conventional relays but not for EH relays.
- By comparing with simulation results, the accuracy of our analysis is validated. Besides, extensive numerical results are presented to acquire insights into the roles of numerous system parameters. Our analytical model can thus be used as an efficient alternative to time-consuming simulations.

The remainder of this paper is organized as follows. Section 2 specifies the considered system settings. Section 3 identifies the conditions under which a one-dimensional DTMC model can be used to capture the evolution of an arbitrary relay battery. Based on the DTMC model, Section 4 analyzes the outage probabilities of two variants of AF relays with relay selection. Section 5 presents the numerical result, and Section 6 gives the concluding remarks.

2. SYSTEM CONFIGURATION

2.1. Network topology and channel model

A two-hop cooperative relaying network is considered, where one source node $S$ relies on a set of randomly distributed nodes $\mathcal{R} = \{R_1, \ldots, R_N\}$ serving as the potential relays to forward its signal to a destination node $D$. 


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expressed as loss with exponent \( \alpha \). Hence, the instantaneous SNR can be expressed as
\[
\Gamma_{ij} = y_{ij} \Omega_{ij} \tag{1}
\]
where
\[
y_{ij} = \frac{P_t}{N_0 d_{ij}^{-\alpha}} \tag{2}
\]
with \( P_t \) denoting the transmission power of transmitter \( i \), \( N_0 \) the noise power at the receiver, and \( d_{ij} \) the distance between transmitting node \( i \) and receiving node \( j \). Finally, as commonly assumed in the literature, nodes are synchronized, and time is divided in slots of equal length.

2.2. Operation of energy harvesting relays and battery model

In the considered network, each relay is equipped with an energy harvester that can pick up RF signal to power up circuits. The harvested energy is converted to a DC voltage and then stored in a battery for later use. The size of relay battery, denoted as \( B \), is identical to all relays. Besides, the energy conversion efficiency is assumed to be a constant \( \kappa \in [0, 1] \) for all relays. Hence, the amount of energy that can be stored in the battery of relay \( R_n \) per source transmission is equal to \( E_n = \kappa P_S \Omega_{SR} d_{SR}^{-\alpha} \). Notice that the terms “energy” and “power” are used exchangeably based on the normalized time slot. We ignore the energy consumed for signal processing as it is often much smaller than that for transmission [5,13].

In tracking the relay battery status, a discrete battery model as in [11] is adopted by partitioning the battery size into \( L \) levels according to a certain partition threshold. For simplicity, we assume a uniform partition where each battery level has the same width. Denote \( \epsilon_l \) the \( l \)th partition threshold, which can be given as
\[
\epsilon_l = \begin{cases} 
0, & l = 0 \\
\frac{IB}{L}, & l \in [1, L] \\
B, & l = L + 1
\end{cases} \tag{3}
\]
Under the discrete battery model, denote \( \tilde{L} \) as the battery level corresponding to the relay transmission power \( P_R \).

2.3. Protocol and relay selection rule

A two-phase cooperative relaying protocol with proactive relay selection is considered and described in the following. Prior to the source transmission, a best relay \( R^* \) that maximizes the e2e SNR is selected to carry out information relaying. Then the source starts to broadcast its signal during phase I. In the meantime, the selected relay \( R^* \) overhears the source signal, while the rest relays perform EH in phase II. In phase II, relay \( R^* \) forwards the amplified signal to the destination node that completes one cooperation round. Although the relay nodes not selected in phase I can also harvest energy inherent in the relay transmission, the analysis conducted in the following assumes these relay nodes are not active in phase II. This allows us to concentrate on the EH gain provided by source transmission.

According to the aforementioned description, a relay might be in one of three operational modes.

- Forwarding mode: A relay is selected and has sufficient energy to transmit. In this case, the selected relay overhears the source transmission in phase I but can not harvest energy because of the energy half-duplex constraint.
- Harvesting mode: A relay not selected for transmission will perform EH during phase I and stay inactive in phase II.
- Idle mode: A selected relay finds itself short of required energy to forward the source message. As a result, this relay will be idle in phase II.
3. EVOLUTIONS OF RELAY BATTERY

For EH relays with finite energy storage, their feasibilities in assisting the source transmission depend on the individual battery status. For a particular relay, its battery level may increase, decrease, or remain unchanged if it is in the harvesting mode, forwarding mode, or the idle mode. Furthermore, a relay is in the forwarding mode if other relays with sufficient energy to transmit are not selected. Because of this dependency, exact modeling for the relay battery is highly complex particularly when the number of relays is not known a priori. In this section, we first identify some important assumptions that allow for analytically tractable modeling as presented in the second part of this section.

3.1. Assumptions for analysis tractability

Based on the aforementioned system model, the evolution of the relay battery can be modeled as DTMC, where each state corresponds to the battery status of the relay node at the relay selection epoch. This method has been used in [11] and [12] for the scenario of a single relay. In the presence of multiple relays with relay selection, the state transition in the DTMC becomes involved because whether a node is selected will not only affect its own charging/discharging behaviors but also others. As a result, one needs to track the battery status for every relay that exponentially increases the number of states in the DTMC.

The DTMC can be simplified if each node has an equal chance to be selected such that the dependency between the charging/discharging behaviors of different relays can be resolved. Being equally likely selected is also beneficial to EH relays because otherwise one or few relays that are more frequently selected may drain their batteries faster and thus cannot assist the source transmission. Specifically, the e2e SNRs of relay nodes are dominated by the path loss, which exponentially attenuates the received signal strength with the distance. As a result, the SNR maximization selection rule will favor those relays that are closer to the source or the destination. The condition of being equally likely selected holds when the instantaneous SNRs of relays are independent and identically distributed (i.i.d.). In other words, both small-scale and large-scale fading powers of different relay nodes are i.i.d., according to Equation (1). While the former can be justified for the cooperative relaying network because relay nodes are geographically apart from each other, the latter is not valid if relay nodes are stationary, and hence, each relay node suffers deterministic but distinct path loss.

For analytical tractability, we have the following assumptions.

(A1) We use a homogeneous two-dimensional Poisson point process with intensity $\lambda$ [16] to model the initial relay node distribution. According to Poisson point process, the relay nodes are uniformly distributed in an area $A$ with radius $r_0$, and the number of relay nodes $N$ is Poisson distributed with mean measure $\lambda|A|$. On the other hand, the locations of $S$ and $D$ are fixed such that their distance is a constant.

(A2) During consecutive relay selection epochs, the locations of relay nodes change randomly according to the uniform mobility model [17], where each relay node moves in the direction uniformly distributed in $(0,2\pi]$ with a constant speed and a distance following the exponential distribution. It is known that the steady-state distribution of the node positions according to the uniform mobility model remains uniform.

(A3) Relay nodes perform AF to forward the source signal by amplifying the received signal with a certain gain and then forwarding the amplified signal to the destination node. Depending on the choice of the relay gain, AF relays can be classified into fixed-gain AF (FAF) and variable-gain AF (VAF). The former, also known as the CSI-based AF, adjusts the amplification gain based on instantaneous CSI of the source-relay link [18].

(A4) For simplicity, the energy cost for relay selection is not considered in the subsequent analysis. Typically, the data volume and the transmission power for control signaling are constant, and thus, the inclusion of energy consumption for control signaling in the analysis should be straightforward.

We note that assumptions (A1) and (A2) imply that internode distances between the source and relays and between the destination and relays are statistically i.i.d. Hence, the average signal attenuations due to path loss are i.i.d. for all relay nodes. Together with i.i.d. fading, the average probability of each relay being selected is identical using the SNR maximization selection rule. Notice that in practice, node movement may be correlated, and thus, the distribution of node positions are not completely uniform. Yet, the analysis based on the assumption of uniform node distribution helps to identify the performance upperbound of spatially random relays.

3.2. Discrete-time Markov chain modeling

By assuming that each relay has an equal chance to be selected, we can focus on an arbitrary relay and use a simple one-dimensional DTMC to capture the evolution of the relay battery as elaborated in the following. With slotted time, the charging/discharging behavior of an arbitrary relay $R_n$ is a discrete-time stochastic process $X_t$ with the state space $\{0, 1, \ldots, L+1\}$, where $X_t$ represents the energy level at the end of phase $t$ (e.g., $X_0$ represents an empty battery and $X_{L+1}$ represents a full battery). Hence, $\{X_t\}$ can be modeled as a DTMC with $L + 2$ states. The DTMC $\{X_t\}$ is characterized by the state transition probability matrix...
The harvest energy transition probability from state $s_i$ to state $s_j$ and $\phi_i$ can be interpreted as the average probability that an arbitrary relay sees its energy buffer at level $i$ at the selection epoch. In the following, we first derive the state transition probabilities for a given network realization where the number of relays $N$ is fixed. Then we obtain the stationary distribution of the DTMC $\{X_t\}$.

Some notations will be used throughout the analysis. For a random variable $X$, $f_X(x)$ and $F_X(x)$ denote the probability density function and cumulative distribution function (CDF), respectively. The expectation of $X$ is either written as $E[X]$ or $\bar{X}$. If $X$ is exponentially distributed, $F_X(x) = 1 - \exp(-x/\bar{X})$. Also, $\Omega_{S,R,i} \triangleq \Omega_{i,1}$ and $\Omega_{R,D} \triangleq \Omega_{N,2}$.

### 3.2.1. $s_0 \rightarrow s_j$ ($0 \leq j < L + 1$).

An empty relay battery is partially charged to level $j$ if the harvest energy $E_R = P_s \Omega_{N,1} d^2_{SR,j}$ falls in between $\epsilon_j$ and $\epsilon_{j+1}$ with probability

$$p_{0,j} = P \left[ (R_n \not= R^*) \cap (\epsilon_j < E_R < \epsilon_{j+1}) \right]$$

$$= P[R_n \not= R^*]$$

$$\times P \left[ \frac{\epsilon_j}{P_K} \leq \Omega_{S,R} < \frac{\epsilon_{j+1}}{P_K} | R_n \not= R^* \right]$$

where Equation (4.a) follows from assumptions (A1) and (A2) such that $P[R_n = R^*] = 1/N$. As to Equation (4.b), we use the fact that $d_{SR,j}$ is an random variable with distribution $f_{d_{SR,j}}(r) = 2r/(\pi r_0)^2$, according to assumption (A1) that relay nodes are uniformly distributed over an area with radius. Thus, we have

$$F_{\Omega_{S,R,j}}(\xi d^2_{SR,j}) = E[d_{SR,j}] \left[ F_{\Omega_{S,R,j}}(\xi d^2_{SR,j}) \right]$$

$$= \int_0^{\pi} \left( 1 - e^{-\xi d^2_{SR,j}} \right) f_{d_{SR,j}}(r) dr$$

where $\xi$ is an arbitrary constant and the integral is solved with the aid of [19, 3.381-8] and $\Gamma(\cdot, \cdot)$ represents the lower incomplete gamma function [19, 8.350.1].

### 3.2.2. $s_0 \rightarrow s_{L+1}$.

This case arises when a relay with an empty energy buffer is fully charged with probability

$$p_{0,L+1} = P \left[ (R_n \not= R^*) \cap (E_R \geq B) \right]$$

$$= \frac{2G(2/\alpha, B/(\pi r_0)^2/P_K \Omega_{S,R})}{(\pi r_0)^2} \left( 1 - \frac{1}{N} \right)$$

(6)

which is obtained in the same manner as Equation (4).

### 3.2.3. $s_j \rightarrow s_0$ ($0 \leq i < l + 1$).

A relay battery status does not change across two consecutive cooperation runs in two cases. (i) The tagged relay is in the harvesting mode but the harvested energy is not enough to increase its battery level, or (ii) the tagged relay is in the idle mode because its current battery level $s_i$ is less than the required transmission power level $L$. If $i \geq L$, only case (i) will occur. On the other hand, when $i < L$, case (i) occurs if the relay is not selected, while case (ii) takes place because the relay is selected, but it is short of enough power to transmit. Hence, the associated transition probability is given by

$$p_{i,0} = P \left[ (R_n \not= R^*) \cap (\epsilon_j \leq \epsilon_i + E_R < \epsilon_{j+1}) \right]$$

(7)

where $i \geq L$, and

$$p_{i,0} = P \left[ R_n = R^* \right]$$

$$+ P \left[ (R_n \not= R^*) \cap (\epsilon_i \leq \epsilon_j + E_R < \epsilon_{j+1}) \right]$$

(8)

if $i < L$. After some manipulations, $p_{i,0}$ can be obtained as

$$p_{i,0} = \left\{ \begin{array}{ll} \frac{2}{(\pi r_0)^2} \frac{G(2/\alpha, \epsilon_j/(\pi r_0)^2/P_K \Omega_{S,R})}{\alpha(\epsilon_j/\Omega_{S,R})^{2/\alpha}}, & i \geq L \\ \frac{1}{N} + \frac{2}{(\pi r_0)^2} \frac{G(2/\alpha, \epsilon_j/(\pi r_0)^2/P_K \Omega_{S,R})}{\alpha(\epsilon_j/\Omega_{S,R})^{2/\alpha}}, & i < L \end{array} \right.$$

(9)

### 3.2.4. $s_l \rightarrow s_j$ ($0 \leq j < l + 1$).

The battery level of a relay reduces from $i$ to $j$ occurs only when this relay is in the forwarding mode. Hence, we have

$$p_{ij} = \left\{ \begin{array}{ll} \frac{1}{N}, & i \geq L \\ 0, & i < L \end{array} \right.$$

(10)

### 3.2.5. $s_l \rightarrow s_{L+1}$ ($0 < i < L + 1$).

A partially charged relay battery becomes fully charged only if the relay is in the harvesting mode. Thus, the transition probability from state $s_l$ to state $s_{L+1}$ can be computed as

$$p_{L+1} = P \left[ (R_n \not= R^*) \cap (\epsilon_i + E_R \geq B) \right]$$

$$= \frac{2G(2/\alpha, (B-\epsilon_i)/(\pi r_0)^2/P_K \Omega_{S,R})}{(\pi r_0)^2} \left( 1 - \frac{1}{N} \right)$$

(11)
3.2.6. \( s_i \rightarrow s_j (0 < i < j < L + 1) \).

When a relay with a non-empty battery at level \( i \) is in the charging mode, its battery remains partially charged with probability

\[
p_{ij} = \mathbb{P} \left[ \left( R_n \neq R^* \right) \cap (e_j \leq b_i + E_n < e_{j+1}) \right] \]

\[= \frac{2}{(n)^2} \left[ \frac{G(2/\alpha, (e_{j+1} - e_i)/(P_\alpha\kappa\Omega_S R_\alpha))}{(e_{j+1} - e_i)/P_\alpha\kappa\Omega_S R_\alpha} \right. \]

\[- \left. \frac{G(2/\alpha, (e_j - e_i)/(P_\alpha\kappa\Omega_S R_\alpha))}{(e_j - e_i)/P_\alpha\kappa\Omega_S R_\alpha} \right](12) \]

\[\times \left( 1 - \frac{1}{N} \right) \]

3.2.7. \( s_{L+1} \rightarrow s_{L+1} \).

A full battery of a relay will remain full in two cases. If \( L + 1 > \bar{L} \), the battery level remains unchanged if the relay is not selected with probability \( 1 - 1/N \). On the contrary, if \( L + 1 < \bar{L} \), the relay will be idle with probability one. Hence, we have

\[p_{L+1,L+1} = \begin{cases} 1 - \frac{1}{N}, & L + 1 \geq \bar{L} \\ 1, & L + 1 < \bar{L} \end{cases} \tag{13} \]

Given the aforementioned transition probabilities, we can find the steady-state probabilities of each state as follows. Firstly, one can verify that the DTMC \( \{X_i\} \) with transition probability matrix \( P \) is homogeneous and row stochastic. Secondly, \( \{X_i\} \) is irreducible because each state is reachable from any other states in finite time with non-zero probability. Furthermore, there is a non-zero probability of staying in an arbitrary state \( s_i \) if the initial state is \( s_i \), according to Equations (4), (7), and (13). Therefore, the DTMC \( \{X_i\} \) is aperiodic. Because \( \{X_i\} \) is homogeneous, irreducible, and aperiodic, a unique steady-state probability vector exists and can be found by solving a set of balance equations \( \phi P = \phi \) as well as the normalization equation \( \sum_{i=0}^{L+1} \phi_i = 1 \). Alternatively, \( \phi \) can be solved as \( \phi = (P^T - I - B)^{-1}b \) where \( I \) is the identity matrix, \( B \) is a \((L+2)\times(L+2)\) matrix with all ones, and \( b = (1, 1, \cdots, 1)^T \) [11].

4. PERFORMANCE OF RELAY SELECTION

In the previous section, we obtain the steady-state probabilities of the relay battery for a given network realization. In this section, we derive the outage probability of relay selection based on the SNR maximization selection rule considering all possible network realizations. We first derive the CDF of the e2e SNR associated with the selected relay for both VAF and FAF relays. Then we present the general expression of the outage probability.

4.1. Cumulative distribution function of end-to-end signal-to-noise ratio

For VAF, the e2e SNR of relay \( R_n \) can be shown as [18, Equation (5)]

\[\Gamma_n^{VAF} = \frac{\Gamma_{S,R_n}\Gamma_{R_n,D}}{\Gamma_{S,R_n} + \Gamma_{R_n,D} + 1} \tag{14}\]

As to FAF relays, the e2e SNR is given by [18, Equation (6)]

\[\Gamma_n^{FAF} = \frac{\Gamma_{S,R_n}\Gamma_{R_n,D}}{C + \Gamma_{R_n,D}} \tag{15}\]

where \( C = P_{R_n}/(G^2 N_0) \) is a constant associated with the fixed-gain \( G \). Denote \( \Gamma^* \) the e2e SNR associated with the selected relay, that is, \( \Gamma^* = \max_k \Gamma_n^{\text{type}_k} \) where type \( \{\text{VAF, FAF}\} \).

The complex forms of the e2e SNR in Equation (14) for VAF and Equation (15) for FAF as well as the random locations of relays render the closed-form expression for the distribution of \( \Gamma^* \) difficult to derive. Alternatively, [20] provides a numerically tractable approach to obtain the CDF of \( \Gamma^* \) considering VAF. We extend the results in [20] to the case of FAF in Appendix, where the conditional outage probabilities for FAF and VAF are indicated in Equations (A.6) and (A.8), respectively.

4.2. Outage probability

The outage event arises when the e2e SNR via the selected relay is below a prescribed SNR threshold \( T = 2^{2\epsilon-1} \) [21], where \( \epsilon \) b/s/Hz is the transmission rate requirement. From the protocol description, the outage event happens in three cases.

- There are no available relays to help. From assumption (A1), the number of relays \( N \) follows the Poisson distribution and thus

\[\mathbb{P}_0 = \mathbb{P}[N = 0] = e^{-\lambda |A|} \tag{16}\]

- When the best relay is in the idle mode with probability \( \mathbb{P}_I \) given by

\[\mathbb{P}_I = \sum_{i=0}^{\bar{L}-1} \phi_i \tag{17}\]

where \( \bar{L} \) is the battery level corresponding to \( P_R \) and \( \phi_i \) is the steady-state probability of the battery status obtained in Section 3.2.

- The best relay is in the forwarding mode, but its end-to-end SNR \( \Gamma^* \) falls below \( T \) with probability \( \mathbb{P}_{out,F} \) as given by
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\( P_{out,F} = P \left[ \Gamma^* < T | R^a \right] \times P \left[ R^a \text{ in the forwarding mode} \right] \)
\[ = P \left[ \Gamma^* < T \right] \times \sum_{l=1}^{L} \phi_l \]
\[ \text{where } P \left[ \Gamma^* < T \right] = F_{P,T}(T) \text{ can be obtained by Equations (A.5) and (A.7) for FAF and VAF, respectively.} \]

Overall, the outage probability can be expressed as
\[ P_{out} = P_0 + P_1 + P_{out,F}. \]

It is noted that for a sufficient large area (i.e., the radius of \( A \) is much larger than \( d_{S,D} \)), \( P_0 \) is relatively small and may be negligible. For example, when the radius of \( A \) is 5 m and \( \lambda = 0.3, P_0 \approx 10^{-11} \).

5. NUMERICAL RESULTS

Simulation results are presented to evaluate the performance of the cooperative scheme described in Section 2.3 for both VAF and FAF. We also verify the accuracy of our theoretical analysis in this section. Unless specified, we set \( r_0 = 10 \text{m}, d_{S,D} = 7 \text{m}, \) path loss exponent \( \alpha = 4, \) transmission rate \( R = 0.5 \text{bps/Hz}, \) the number of energy levels \( L = 10, \) and energy conversion efficiency \( \kappa = 0.6. \)

Besides, all relays have the identical transmission power and average fading power, that is, \( P_S = P_R = P, \Omega_{S,R}, = \Omega_{R,D} = 1. \) The battery size \( B \) is set to be a multiple of \( P, \) that is, \( B = \beta \cdot P, \) where \( \beta > 0 \) is referred to as the battery scaling factor. In all figures, the term “SNR” represents \( P'/N_0. \)

Figure 2 compares the outage probabilities of VAF and FAF with relay density \( \lambda = 0.2, \) battery scaling factor \( \beta = 10 \) and 20, and the relay amplification gain \( G^2 = 10 \) for FAF. It can be seen that the analytical results match to the simulated ones well, confirming the accuracy of our analysis. As expected, the outage probabilities of both VAF and FAF relays decrease with SNR. We note that the outage curves associated with FAF and VAF are close to each other at the low SNR region, and they intersect at a certain SNR value. The same pattern is also reported in [18] with relays supplied by fixed power sources. We note that the impact of battery size to the outage probability is quite different for FAF and VAF. As the battery scaling factor increases from 10 to 20, the outage probability of FAF remains invariant, while that of VAF dramatically decreases particularly at high SNR. On the other hand, there exhibits an error floor when VAF is used with a relatively small battery (\( \beta = 10 \)). To explain this, recall Equation (19) that the outage probability is a function of the idle probability \( P_1 \) and the conditional outage probability \( P_{out,F} \), and they are plotted in the next figure.

Figure 3 depicts the outage probability of VAF versus SNR under the same setting as Figure 2 with varied battery scaling factors. The idle probability \( P_1 \) with \( \beta = 10 \) and the conditional outage probability \( P_{out,F} \) with \( \beta = 20 \) are also plotted. When the battery size is relatively small (\( \beta = 10 \)), one can see that the outage probability of VAF is dominated by the idle probability at the high SNR region, which explains the cause of the error floor. As the battery size increases (\( \beta = 20 \)), the idle probability is infinitely small such that the outage probability is primarily determined by \( P_{out,F} \), which remains static as the battery size further increases (\( \beta = 30 \)). As a result, the outage probability does not change with the battery size when it is sufficiently large. The aforementioned results suggest that the battery size matters for VAF but not for FAF.

Next, we investigate the impact of energy conversion efficiency \( \kappa \) to the outage performance in Figure 4 under different SNRs with \( \lambda = 0.5, \beta = 20, \) and \( G^2 = 10 \) for
The interaction of outage probability and the relay density is studied in Figure 5, where only FAF is shown because VAF reveals the same trend and thus is omitted. In the figure, we vary the relay gain $G^2$ and fix $\beta = 20$ and signal-to-noise ratio (SNR) $= 20$ dB. Clearly, the analytical results match with the simulated ones well. The outage curve is a negative exponential function of the relay density $\lambda$. The exponential decay is much faster when $\alpha$ drops from 3 to 2 than from 4 to 3. Take $\lambda = 0.2$, for example, the outage probability with $\alpha = 3$ is $10^4$ times than that with $\alpha = 4$, and over $10^7$ times than that with $\alpha = 2$. The impact of path loss is more pronounced as the relay density increases. The aforementioned results reveal the remarkable impact of propagation loss because of the distance to the performance of cooperative relaying based on randomly deployed EH relays.

6. CONCLUSIONS

Because the exact analysis for opportunistic relay selection using EH relays is not tractable, this work presented an alternate analysis under the hypothesis that the relay SNR $= 20$ dB. The accuracy of our theoretical analysis is confirmed by the reasonable match between analytical and simulation results. It can be seen that the outage probability decreases with $\lambda$, and it shows an exponential decay as the density $\lambda$ increases (an exponential decay curve is linear on the logarithmic axis). This implies that randomly deployed EH relays is capable of offering the spatial diversity gain. We note that the similar result has been reported in [22], which considers relays with fixed power supply. Here we can see that at SNR $= 20$ dB, a higher relay amplification gain results in a faster decay rate of the outage probability with respect to the relay density, corresponding to a higher spatial diversity gain. However, the decay rate difference between $G^2 = 10$ and $G^2 = 20$ is relatively minor compared with that between $G^2 = 2$ and $G^2 = 10$. Hence, a moderate relay amplification gain should be sufficient to fully exploit the spatial diversity based on EH relays.
positions change independently and uniformly from one selection epoch to the next. With this hypothesis, the analysis is tractable, and we are able to obtain analytical expressions for the outage probability of both VAF and FAF relays using the SNR maximization selection rule.

Through numerical results, we validate the analysis accuracy and acquire insights into the SNR maximization relay selection rule with randomly distributed EH relays. Firstly, we see that the outage probability exponentially decays with both relay density and path loss exponent. Secondly, it is interesting to observe that for EH relays based on FAF, the outage probability is sensitive to the choice of the relay gain but not to the battery size and the energy conversion efficiency. On the contrary, VAF shows an error floor when the battery size is small, but it performs much better than FAF when the battery size is large enough. However, the gain of VAF over FAF is achieved with the frequent update of CSI to properly adjust the amplification gain, which requires extra power consumption to send training sequence and feedback. The analysis conducted in this work provides a convenient assessment tool to determine the relay density and the choice of FAF or VAF according to the practical requirements. Some further extensions of the present work include the consideration of different relay selection rules and the inclusion of other renewable energy sources to enhance the efficacy of EH relays.

**APPENDIX**

To derive the CDF of $\Gamma^*$, we use the approach proposed in [20], which focuses on VAF relays. Here, we extend [20] to the case with FAF relays. According to the channel model given in Equation (1), the e2e SNR is a joint function of relay locations and fading powers. For a specific relay location, the path loss is a constant and so is $\gamma_{i,j}$ in Equation (2). Hence, the condition CDF of $\Gamma_n$ given in Equation (15) can be expressed as

$$F_{\Gamma_n}^{\mathrm{FAF}}(x|p,q) = \mathbb{P}\left[\frac{p\Omega_{S,R_n}q\Omega_{R_n,D}}{C + q\Omega_{R_n,D}} < x\right]$$  \hspace{1cm} (A.1)

where $p \triangleq \gamma_{S,R_n}$ and $q \triangleq \gamma_{R_n,D}$. Equation (A.1) can be derived as

$$F_{\Gamma_n}^{\mathrm{FAF}}(x|p,q) = \int_0^\infty \mathbb{P}\left[\Omega_{S,R_n} < \frac{(py + C)}{pqy}\right] \Omega_{R_n,D} = y\right) dy$$

$$= \frac{1}{\Omega_{R_n,D}} e^{-\frac{y}{\Omega_{R_n,D}}} \int_0^\infty e^{-\frac{y}{\Omega_{R_n,D}}} \frac{1}{\Omega_{R_n,D}^\gamma} dy$$  \hspace{1cm} (A.2)

where the integral can be solved using [19, 3.324–1]. Thus, we have

$$F_{\Gamma_n}^{\mathrm{FAF}}(x|p,q) = 1 - \frac{2}{\Omega_{R_n,D}} e^{-\frac{y}{\Omega_{R_n,D}}} \frac{C\Omega_{S,R_n,D}}{pq\Omega_{R_n,D}^\gamma}$$

$$\times K_1\left(2\sqrt{\frac{C\Omega_{S,R_n,D}}{pq\Omega_{R_n,D}^\gamma}}\right)$$  \hspace{1cm} (A.3)

where $K_1(\cdot)$ is the first-order modified Bessel function of the second kind. By averaging all realizations of the relay locations, the CDF of $\Gamma^*$ for VAF relays can be obtained as [20]

$$F_{\Gamma^*}^{\mathrm{FAC}}(x) = \mathbb{E}\left[\prod_{R_n \in R} F_{\Gamma_n}^{\mathrm{FAF}}(x|p,q)\right]$$

$$= \exp\left(-\lambda \int_\mathcal{A} \left[1 - F_{\Gamma_n}^{\mathrm{FAF}}(x|p,q)\right] ds\right)$$  \hspace{1cm} (A.4)

where $ds$ is the surface element. Substituting Equation (A.3) into Equation (A.4) and using biangular coordinates as in [20], we can solve the integrand in Equation (A.4) and then obtain the outage probability as

$$F_{\Gamma^*}^{\mathrm{FAC}}(x) = \exp\left(-\lambda d_{SD}^2 D_{\mathrm{FAF}}(\gamma_s, \gamma_d, \tilde{\Omega}_1, \tilde{\Omega}_2, \alpha, x)\right)$$  \hspace{1cm} (A.5)

where $\gamma_s = \frac{P_S}{\left(N_0 d_{SD}^2\right)}$, $\gamma_d = \frac{P_R}{\left(N_0 d_{SD}^2\right)}$, and

$$D_{\mathrm{FAF}}(\gamma_s, \gamma_d, \tilde{\Omega}_1, \tilde{\Omega}_2, \alpha, x) = \frac{4}{\tilde{\Omega}_2} \int_0^\pi \int_0^{\pi - \theta_d} \exp\left(-\frac{x}{\tilde{\Omega}_1 \gamma_d^{\frac{1}{2}}} \sin^\gamma(\theta_d)\right)$$

$$\times \frac{\sqrt{\tilde{\Omega}_1 \gamma_d^{\frac{1}{2}}} \sin^\gamma(\theta_d) \sin^\gamma(\theta_s)}{\sin^\gamma(\theta_d + \theta_d)}$$

$$\times K_1\left(2\sqrt{\frac{\sin^\gamma(\theta_d) \sin^\gamma(\theta_s)}{\tilde{\Omega}_1 \tilde{\Omega}_2 \gamma_d}}\right) d\theta_d d\theta_s$$  \hspace{1cm} (A.6)

where $C$ is defined in Equation (15), $\tilde{\Omega}_1 = \tilde{\Omega}_{S,R}$ and $\tilde{\Omega}_2 = \tilde{\Omega}_{R,D}$. Notice that the relay index $n$ has been dropped because of the i.i.d. assumption.

Following the same principle, the CDF of $\Gamma^*$ for VAF relays can be obtained as

$$F_{\Gamma^*}^{\mathrm{VAC}}(x) = \exp\left(-\lambda d_{SD}^2 D_{\mathrm{VAF}}(\gamma_s, \gamma_d, \tilde{\Omega}_1, \tilde{\Omega}_2, \alpha, x)\right)$$  \hspace{1cm} (A.7)

where
\[ \mathcal{D}^{\text{YAF}}(\gamma_s, \gamma_d, \Omega_1, \Omega_2, \alpha, \chi) = 4 \int_{\theta_s}^{\pi} \int_{0}^{\pi - \theta_s} \exp \left( - \frac{\sin^2(\theta_d) + \sin^2(\theta_s)}{\sin^2(\theta_d + \theta_s)} \right) \times \frac{x^2 + x}{\Omega_1 \Omega_2 \sin^2(\theta_d) \sin^2(\theta_s)} \frac{1}{\sin^3((\theta_s + \theta_d))} \, d\theta_d \, d\theta_s \] 

We note that (A.6) and (A.8) involve double integral that cannot be further simplified but can be evaluated numerically.

**REFERENCES**


AUTHORS’ BIOGRAPHY

Kuang-Hao (Stanley) Liu received the PhD degree in Electrical and Computer Engineering from the University of Waterloo, Canada, in 2008, respectively. From 2000 to 2002, he was a software engineer in Siemens Telecom System Ltd., Taiwan. From 2004 to 2008, he was a research assistant in the Broadband Communications Research Group at the University of Waterloo, Canada. He is currently an associate professor in the Department of Electrical Engineering, National Cheng Kung University, Tainan, Taiwan. His recent research focuses on cover cooperative communications, energy-efficient communications, software-defined radios, and beyond 4G technologies. Dr Liu has been participating in organizing several international conferences, including Chinacom’09 and ’10, Wicon’10, IEEE PIMRC’12, and IEEE SmartGridComm’12. He has served as a technical program committee member for many IEEE conferences, such as the IEEE International Conference on Communications and the IEEE Global Telecommunications Conference. He was the guest editor for the IET Communications (special issue on Secure Physical Layer Communications). He is now the editor for the IEEE Wireless Communications Magazine. He is the recipient of the Best Paper Award from the IEEE Wireless Communications and Networking Conference 2010.